# In or Out: Faculty Research and Consulting

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**Abstract**

We examine university-industry knowledge flows in the context of faculty consulting. Our model incorporates faculty decisions to work on their own university research projects or on a project in a firm lab. In equilibrium, faculty research and funding are functions of faculty quality, project characteristics, the faculty share of license revenue from university research, and R&D spillovers. We exploit a unique database of university research funding, publications, and patents for 458 faculty inventors to estimate the parameters of the model. The most novel empirical results are that government research funding is positively related to consulting, a result that can only occur in the theoretical model in the presence of spillovers from the faculty member's university research to the firm. We also find that government and industry funding with the university act as strategic complements.

## 1 Introduction

While universities clearly contribute to industrial innovation, there are important gaps in our understanding of the mechanisms involved (Adam Jaffe 1989, Jerry Thursby and Marie Thursby 2006). Much of our understanding comes from the analysis of spillovers associated with publications or patents (James Adams 1990, Jaffe, Manuel Trajtenberg, and Rebecca Henderson 1993). Except for studies of licensing or start-ups from universities, there is little modeling of

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the actual mechanisms behind such spillovers (Lynne Zucker, Michael Darby, and Maryanne Brewer 1998, Thursby and Thursby 2007). Nonetheless, the few studies that examine informal mechanisms, such as consulting, find that industrial managers often consider these to be more important than either patents or licensing (Wesley Cohen, Richard Florida, Lucien Randazzese, and John Walsh 1998). Moreover, as shown by Edwin Mansfield (1995), understanding consulting requires understanding faculty decisions to work on industry funded projects both within their universities and in company labs, as well as their ability to obtain government research funding.

This paper makes two contributions. First, we develop a model of consulting which incorporates faculty decisions to conduct research within the university or outside in a firm's lab as well as the decisions of funding agents, both government and industrial, on support for the researcher's work within the university. Second, we exploit a unique database of funding, publications, and patents for 458 faculty inventors to estimate parameters of the model.

The model has two stages. In the first stage, a government funding agency and firm simultaneously choose funding levels for a researcher's university research project. This stage is followed by another simultaneous-move game in which the firm chooses a unit consulting fee, and the faculty researcher decides how much time to consult for the firm on its project. The model yields predictions for the time spent consulting and the associated fee, as well as the level of government and industry support for university research.

We allow for the fact that researchers vary in quality or academic reputation, as well as differences in the scientific merit of projects within the university and firm. Research on both projects is uncertain. The firm can benefit from university research in several ways. It can license results from a successful university research project, but regardless of success or whether the firm funded university research projects, it can benefit from the researcher's expertise if it hires her to consult. Thus we allow for R&D spillovers in the sense that the researcher's work on government funded research can enhance her probability of success in the firm's consulting project. We also incorporate the notion that firm funding for research within the university may be more focused or restrictive than government funding. Finally, the university provides some base level of funding for the researcher's internal research. These features allow us to relate consulting behavior to faculty quality, project characteristics, the researcher's share of license revenue from the university project, R&D spillovers, university support for the researcher's internal project, as well as the willingness of the firm and government to sponsor the faculty member's research within the university.

The faculty researcher cares about reputation as well as income, so that the amount of time that she is willing to consult can be increasing or decreasing in the fee depending on whether the she views reputation and income as complements or substitutes. Although some of the model's predictions depend on this relationship, several results hold regardless of the researcher's consulting supply function. In particular, increases in the researcher's quality, university support for the researcher's internal project, or the researcher's share of license revenue from her university research lead to a greater consulting fee. By contrast, increases in the restrictions the firm places on university research funding, R&D spillovers, or the scientific merit of the firm's project all lead to a lower fee. We also find that an increase in restrictions placed by the firm on its university funding increases the amount of time spent consulting, while an increase in the researcher's share of license revenue decreases the time spent consulting.

The effects of government and firm funding for the university project are among those that depend on the slope of the researcher's consulting supply function. However, in the absence of R&D spillovers, an increase in government funding reduces the time spent consulting regardless of the slope. As discussed below this result turns out to be useful in our estimation of the consulting stage.

In the funding stage, obtaining unambiguous results requires additional assumptions in large part because of the ambiguous effects of funding on consulting in the presence of spillovers. Thus, in general, government and firm funding for research within the university can be strategic substitutes or complements. Nonetheless, the additional assumptions needed to characterize this stage are intuitive. For example, if an increase in either type of funding increases the marginal effect of the other on the probability that the researcher's university project will be successful, then we can provide sufficient conditions for government and firm funding to act as strategic complements.

It is important to note that, while we assume the university project is more basic than the firm's, we follow Mansfield (1995) in focusing on consulting projects with some scientific merit. One implication of this is that our work says nothing about so-called contract or routine work for firms. More importantly, it is this feature of our analysis that allows us to overcome a major barrier to examining consulting empirically- that is, a lack of data on consulting time or fees, which to our knowledge are unavailable other than anedotally. While such data are unavailable, we exploit a unique data set of nearly 1690 patents on which 458 faculty from eight major US universities are listed as inventors. Thirty percent of these patents are assigned to (and therefore owned by) firms. In interviews with faculty and university licensing professionals, as well as industry R&D executives, the reason given for faculty patents assigned to firms was consulting (Jerry Thursby, Anne Fuller, and Marie Thursby 2007). We exploit this evidence, and use firm assigned faculty patents as a measure of consulting activity. Admittedly this measure captures only a subset of outcomes from consulting since it ignores consulting that does not lead to patents, but it is useful in the context of our model since projects that result in patents clearly have scientific merit. Our data also include both the industry and government research funding of the faculty as well as their publication and citation records. Thus, we are able to provide estimates for both the consulting and funding stages of the model.

In general, the empirical results support the theory. Results for the consulting stage support our assumption that university research projects are more basic than firm projects as measured by the number of backward citations in university and firm assigned patents and by Manuel Trajtenberg, Adam Jaffe, and Rebecca Henderson's (1997) measure of originality. We also find that in the funding stage, government and industrial funding are strategic complements.

Perhaps the most striking results are those regarding spillovers. In the consulting stage, we find that consulting is positively associated with government funding. In the context of our theoretical model, this result is possible only if there is a spillover from the faculty researcher's government sponsored research to the firm's research problem. As predicted by the theoretical model when government and firm research are strategic complements, we find that industry sponsored research in the funding game is negatively associated with a measure of spillovers that relates firm assigned faculty patents to the faculty member's academic publications.

This paper is one of only a few studies to examine consulting either theoretically or empirically. The aforementioned survey research by Mansfield (1995) and Cohen et al. (1998) is a notable exception. To our knowledge, the only theoretical analyses of consulting are Beath et al. (2003) which examines the potential for budget-constrained universities to relax the constraint by encouraging faculty to consult and Emmanuel Dechenaux, Marie Thursby, and Jerry Thursby 2007) which examines consulting as one of the mechanisms for inducing faculty inventors to collaborate in development needed for commercial success of inventions licensed from the university (Richard Jensen and M. Thursby 2001). The latter differs markedly from this paper since the consulting considered is *ex post* development from a project started in the university rather than *ex ante* research by a faculty member on an industrial project.

The license share result in the theory is of particular note since it contributes to policy debates on the impact of licensing and other commercial opportunities on faculty research. The policy concern is that the opportunity to earn license revenue would divert faculty into applied work or research with little scientific merit. Empirical research examining this issue has been unable to find such an effect and in some cases, has found increased research in response to licensing (Azoulay et al. 2006, 2007, Lach and Schankerman 2004; Thursby and Thursby 2007; and Thursby et al. 2007b). Our theoretical result provides a rationale for these findings since an increased share of license revenue will increase time spent on the university research project. Moreover, if the university research project has more scientific merit than the firm's, the increased share actually leads to an increase in fundamental research.

The work closest to ours is Thursby et al. (2007a) which examines a sample of 5811 patents on which faculty from 87 US universities are listed as inventors. In their sample, 26% of the patents are assigned solely to firms rather than to the faculty member's university. Both their work and ours provide a more nuanced view of academic contributions to industrial patenting than that provided by citations to patents assigned to universities.[1] Their work differs, however, in that it is purely empirical and focuses on assignment as a function of patent characteristics and university policy rather than individual inventor characteristics or research funding. Consistent with our theoretical result on inventor share, they find that a higher inventor share increases the likelihood

[1] See, for example, Jaffe (1989), Jaffe et al.(1993), Henderson et al. (1998). For a similar point in a European context see Crespi et al.(2006), Geuna and Nesta (2006), and Saragossi and van Pottelsberghe de la Potterie (2003).

of university assignment as compared with assignment to a firm in which the inventor is a principal.

Finally, we contribute to the literature on the relationship between government and industry funding for research which has primarily focused on the complementarity or substitutability of public and private funding of R&D conducted by firms (Paul David and Bronwyn Hall 2000, David, Hall, and Andrew Toole 2000). By contrast, we focus on government and industry funding for research in universities. Our combined theoretical and empirical results provide new insights into the ways in which firms benefit from spillovers from government funding for university research. In particular, our empirical approach of identifying faculty contributions to industrial patenting according to firm assigned patents with faculty inventors shows that spillovers are greater than those identified by the common practice of examining citations in firm assigned patents to university assigned patents. By explicitly modeling consulting as the mechanism involved, we are able to link these spillovers to the levels of research funding.

# 2 Environment

Our goal is to develop a theory to explain observed levels of government and industrial funding, consulting, publications, and assignment of patents among faculty researchers, and how they differ with the quality of the faculty. To this end, we employ Occam's razor and assume one faculty researcher, one firm interested in capitalizing on faculty expertise, and one government funding agency. There are many dimensions on which one can measure researcher quality, but for the purposes of this analysis, we assume it can be characterized by an observable variable $q$ defined on the interval $[0, Q]$ such that higher values of $q$ correspond to greater academic success.

## 2.1 Research Technology

Although there are also many dimensions on which one can categorize research, for the purposes of this analysis it is most useful to think in terms of the pure scientific component of a given research problem. Thus, we assume research problems can be characterized by a variable $x$, defined on the interval $[0, X]$, such that higher values of $x$ correspond to research that has greater scientific merit and is inherently more difficult to solve.

Successfully solving a given research problem can generate multiple outputs of value to the researcher, university, government funding agency, or industrial sponsor. These can be generally thought of as those results of research that contribute to the scientific reputations and commercial payoffs associated with solving the problem, such as publications, citations, patents, and profits. The likelihood that a research project succeeds depends on a number of factors, including the nature of the problem to be solved (how fundamental or basic it is), the quality of the researcher, and the level of funding available.

For simplicity, we think of the researcher as working on a single research problem within the university, which has scientific merit $x_I$, with the possibility of also working outside the university as a consultant on a firm's research problem, which has scientific merit $x_O$, where $x_I > x_O$. While this assumption is not necessary for our results, it is consistent with the bulk of the literature on university industry collaboration (Rosenberg and Nelson 1994 and 1996, David Mowery and David Teece 1996, Nicola Lacetera 2007). Assume that $T$ is the total time available in the period, and that $M$ is the (maximum) amount of time that she can spend consulting, $M < T$.[2] Then the timing of the problem is as follows. If $t$ is the time she contracts to consult with the firm, $t \in (0, M]$, then she spends the first $T - t$ "months" working in the university on her own research project, and the last $t$ months working on the firm's problem in its R&D lab. If she does not consult, $t = 0$, then she works all of the year on her own university research.[3]

Now consider her research funding. As a member of the faculty, she has at least minimal research support $K_I > 0$ from the university for her own project, which she can supplement with sponsored research funds from a government agency, $G$, and/or industrial firm, $F$. Her research on the firm's problem is conducted within the firm's own R&D lab, where $K_O > 0$ is the fixed level of research support provided by the firm in this lab. The unit cost of consulting paid by the firm is $c$, so the researcher is paid $c$ per unit of time for consulting (i.e., $c$ is the unit cost of consulting, so her consulting income is $ct$.

As is common, we model research as an uncertain production process in which the "production function" is a probability of success function. We assume that the probability of success in solving any specific research problem of scientific merit $x$ undertaken by a researcher of quality $q$ is $p(\tau, e; q, x)$, where $\tau$ represents the time the researcher devotes to the project, and $e$ represents her effective funding on that project. From the production perspective, it is natural to assume that $p$ is increasing and strictly concave in ($\tau$, $e$, $q$), so these "inputs" have positive but diminishing marginal productivities. It is also natural to assume that these inputs are complements, so the second order cross-partial derivatives of $p$ with respect to them are all positive. For example, the marginal effect of an additional hour of research on the probability a project will succeed should be greater for researchers with higher quality or greater levels of funding. Our assumption that it is more difficult to solve problems with greater scientific merit implies $p$ is decreasing in $x$, and it also natural to assume that this difficulty increases at a increasing rate, $\partial^2 p/\partial x^2 < 0$. We also assume that a more difficult project reduces the marginal effect on the probability of success

[2] Most funding agencies and universities will not allow researchers to sell more than 100% of their time, so a decision to consult for the firm in its research lab on its project clearly means that the researcher will not be spending all of her time on her university project. Indeed, if she chose to so this after accepting, for example, federal funding for the entire year, then the granting agency would undoubtedly adjust their level of funding for her to adjust for this.

[3] This implicitly assumes that our heroine is an obsessive-compulsive workaholic who prefers her own research to all forms of leisure activity. This interpretation is perhaps an oversimplification, but it highlights the stylized fact that most researchers view their own research as a consumption good.

of time, effective funding, and quality, so that the cross-partial derivatives with respect to $x$ and each of the inputs are negative.

It is important to discuss in more detail what we mean by "effective" research funding. Essentially this concept has been developed to address two commonly observed stylized facts about research funding. First, funding sources typically differ in the types of constraints they place upon the uses of the funds they provide. It is generally conceded that funding from government agencies is "better" than that from industry, at least on average, because government agencies place fewer restrictions on the uses of those funds. Second, there are often spillovers between research projects, arising in this case because experience from basic research can affect the probability of success in applied projects, and vice versa (Mansfield 1995, Zucker et al. 1998).

Although there are several well-known and accepted methods for formalizing these stylized facts in models of R&D (see DeBondt (1997)), the approach we take is to define effective funding. Under the timing assumed, consulting occurs (if at all) at the end of a given period, so the only spillovers possible will be from the university project to the consulting project.[4] Thus, we define effective funding on the researcher's project in the university as

$$e_I = K_I + G + \alpha F \tag{1a}$$

where $\alpha \in (0, 1)$ represents the fraction of industrial funding that is equivalent to government funding. That is, if industrial funding had the same restrictions on its use as government funding, then we would have $\alpha = 1$. However, $\alpha$ decreases as the additional constraints imposed on industrial funding rise. Analogously, we define effective funding for the firm's project as

$$e_O = K_O + \beta G + F + ct, \tag{1b}$$

where $\beta \in [0, 1)$ represents the extent to which her university research experience can contribute to solving the firm's problem. It is worth noting that this structure assumes that industrial funding of basic research projects within universities can be justified not only by the possibility these projects might yield results with commercial application, but also by the possibility that this basic research experience might indirectly affect the firm's own internal research problems.

## 2.2 Preferences and Payoffs

We assume that faculty utility at any date is $U(R, W)$, where $R$ is her current stock of academic (scientific) reputation and $W$ is her current wealth stock. Marginal utility in reputation is positive and diminishing, while marginal utility of wealth is positive and nondecreasing (we allow for the case of risk neutrality to

[4] We do not claim that there are no spillovers from applied to basic research, but rather that in this model any such spillovers would have to emanate from previous consulting projects not incorporated in this model. We abstract from these spillovers because they are not the focus of the analysis.

clarify which results do not depend on risk-aversion). Let $R_s$ denote her reputation if she successfully solves her university research problem in this period. We assume $R_s$ is an increasing function of $x_I$, because successful solution of a research problem of greater scientific merit results in greater enhancement of her reputation.[5] Let $R_f$ denote her reputation if she fails to solve the problem in this period. This is also her reputational stock at the beginning of the period, when the funding agency and firm make their funding decisions. Naturally we assume $R_s > R_f$, so $R_s - R_f$ is the flow of reputation in this stage (conditional on success). To keep the notation as compact as feasible, define $A$ as her wealth stock at the beginning of the period plus her university salary minus savings. Roughly speaking, $A$ is her current net assets (i.e., net of savings and non-innovation income). Also assume that $\gamma$ is her share of the license revenue paid to the university for a success, $L \geq 0$ (we allow $L = 0$, as this is the case for many university research projects). Then current wealth is $A + \gamma L + ct$ for success and $A + ct$ for failure, forms which emphasizes the flow income from university invention and consulting. Therefore, the researcher's expected utility is

$$\begin{aligned} EU(G, F, t, c) &= p(T-t, e_I; q, x_I)U(R_s, A + \gamma L + ct) \\ &\quad + [1 - p(T-t, e_I; q, x_I)]U(R_f, A + ct). \end{aligned} \tag{2}$$

This approach allows us to focus on any given stage in the life cycle of this researcher. From her perspective, the results that follow depend on the stage of the life cycle only to the extent that they depend on the relative magnitudes of $R$ and $W$.[6]

The government funding agency is primarily interested in advancing basic scientific research, so its utility, $U_g$, depends upon the scientific reputational stock associated with the research it has funded. Because there are alternative uses for its research budget, namely other researchers' projects, its net expected utility, $EU_g$, from funding this particular project is the expected utility of its reputation less the utility loss $V$ from not funding alternative projects. Its net expected utility from devoting $G$ to this project is then

$$\begin{aligned} EU_g(G, F, t, c) &= p(T-t, e_I; q, x_I)U_g(R_{gs}) \\ &\quad + [1 - p(T-t, e_I; q, x_I)]U_g(R_{gf}) - V(G) \end{aligned} \tag{3}$$

where $R_{gs}$ is its reputational stock if she succeeds in her university project, and $R_{gf}$ is its reputational stock at the beginning of the period. We also assume $R_{gs}$ is an increasing function of $x_I$. However, the agency does not get reputational

[5] For notational convenience, we do not write this functional dependence explicitly except when necessary.

[6] This approach abstracts from the savings and salary determination decisions, but the additional complexity from endogenizing them would not add anything of value to the analysis. As we show below, the stage of the life-cycle matters only to the extent that varying the relative stocks of R and W over time might change the sign of $\partial^2 U/\partial R \partial W$, and so possibly the slope of her best reply function in the consulting subgame.

credit for her success if it does not fund her: $R_{gs} > R_{gf}$ if and only if $G > 0$. It is worth noting that an increase in consulting time by the researcher unambiguously decreases the funding agency's expected utility by reducing the probability of success in her university project.

Finally, the firm's expected profit arises from both its own research problem and the university research that it funds. We assume a firm does not fund a researcher's university project unless it obtains an option to license a success from that project. Let $\pi_I$ denote firm profit from funding the researcher's university project if it succeeds, and $\pi$ denote the profit from its own research project if it succeeds. Then its expected profit is

$$E\Pi(G, F, t, c) = p(T-t, e_I; q, x_I)(\pi_I - L) - F + p(t, e_O; q, x_O)\pi - ct. \quad (4)$$

Note that this form implicitly assumes that, if the firm is not interested in funding the research in exchange for a license option, then it would not be interested in a license from a success developed without its funding.

To save on notation, in the following we shall let $p_I$ denote $p(T-t, e_I; q, x_I)$ and $p_O$ denote $p(t, e_O; q, x_O)$ whenever we can do so without causing confusion.

# 3 The Funding Game

Our objective is to develop a game structure that conforms to the stylized fact that faculty typically prefer their own research to consulting, and therefore they focus on obtaining funds for their research before making any agreements to consult. Thus, the game as we envision it has two stages. In the first stage, our heroine seeks support for her university research project from both the government funding agency and the firm. The agency and the firm then simultaneously choose funding levels for the researcher's university project. Then, after these decisions are made and revealed, that is followed immediately (i.e., before the success or failure of the university research project is observed) by another simultaneous-move game in which the firm chooses a unit consulting fee, and the researcher decides how much time to spend consulting for the firm.[7]

Two comments about this approach are in order. First, it assumes that the funding agency and firm must pre-commit to providing funds for the researcher's university project.[8] It also assumes that researchers cannot be treated as agents who must accept take-it-or-leave-it offers. That is, we are interested in modeling the behavior of those "star" scientists whose expertise gives them more "market power" than workers in a principal-agent model with a perfectly elastic supply of labor, an assumption which is unrealistic for star scientists.

[7] This approach also conforms to the "standard" academic year of nine months in which faculty are paid by the university, followed by three summer months in which faculty are free to pursue external funding options.

[8] This approach is similar to that in Lacetera (2005), who assumes that firms commit to university research as a way of funding basic research.

## 3.1 Stage Two Equilibrium

As usual, we begin by considering the second stage game, in which the researcher chooses her consulting time $t$ and the firm chooses its unit consulting fee $c$, given the values of funding for university research chosen in stage one, $F$ and $G$. Firms that devote funds to R&D typically have some ability to adjust their budgets, at least in principle. However, generally such a firm allocates a fixed amount $B_f > 0$ to R&D, and does not make major adjustments until the next budget cycle. Therefore, we assume $c \in [0, B_f/M]$.

**Theorem 1** *Consider the strategic form game with the researcher and firm as the players, whose strategies are $t \in [0, M]$ and $c \in [0, B_f/M]$, and payoff functions are defined by (2) and (4). Also assume each player's payoff function is continuous and strictly quasi-concave in its own strategy, given any strategy choices by the other players. Then this game has a Nash equilibrium $(t^*(G, F), c^*(G, F))$.*[9]

As is well known, under the conditions of this theorem, choosing $t \in [0, M]$ to maximize $EU(G, F, t, c)$ yields a best a best reply function $\hat{t}(c)$ for the researcher[10], which gives the consulting time that maximizes her expected utility for any unit consulting fee chosen by the firm. Similarly, choosing $c \in [0, B_f/M]$ to maximize $E\Pi(G, F, t, c)$ yields a best a best reply function $\hat{c}(t)$ for the firm[11], which gives the unit consulting fee that maximizes its expected profit for any time in consulting chosen by the researcher. The possible equilibria of this game are more easily understood using diagrams of these best reply (or reaction) functions.

Because we are interested in deriving testable implications, we focus on the Nash equilibrium when it is interior, $t^* \in (0, M)$ and $c^* \in (0, B_f/M)$.[12] In this case it must satisfy

$$\frac{\partial EU(G, F, t^*, c^*)}{\partial t} = 0, \tag{5a}$$

and

$$\frac{\partial E\Pi(G, F, t^*, c^*)}{\partial c} = 0 \tag{5b}$$

where

[9] These equilibrium values are also functions of all the parameters of the model ($\alpha$,$\beta$,$q$,$x_I$,$K_I$,$x_O$,$K_O$,$S$,$L$,$\gamma$). Although a minor abuse of notation, we omit these as arguemnts of the functions for clarity of exposition.

[10] We omit the parameters of the model as explicit arguemnts of this function for clarity of exposition.

[11] We omit the parameters of the model as explicit arguemnts of this function for clarity of exposition.

[12] Of course, these results provide some information about corner solutions as well. For example, a change that increases consulting time in an interior equilibrium is more likley to induce a researcher off the no-consulting corner and begin some consulting.

$$\frac{\partial EU(G,F,t,c)}{\partial t} = -\frac{\partial p_I}{\partial \tau}[U(R_s, A+\gamma L+ct) - U(R_f, A+ct)] \tag{6a}$$
$$+[p_I\frac{\partial U(R_s, A+\gamma L+ct)}{\partial Y} + (1-p_I)\frac{\partial U(R_f, A+ct)}{\partial Y}]c$$

and

$$\frac{\partial E\Pi(G,F,t,c)}{\partial c} = \frac{\partial p_O}{\partial e_O}(t\pi) - t = (\frac{\partial p_O}{\partial e_O}\pi - 1)t. \tag{6b}$$

In this case, the best replies are implicitly defined by ($6a$) and ($6b$).

Examples of this equilibrium are depicted in Figures 1 and 2. To interpret the equilibrium conditions in (5), consider the expressions for marginal utility and marginal profit in (6). From ($6b$), an increase in the consulting fee increases effective funding $e_O$, and therefore increases the probability of success in the firm's project and its expected profit, so it increases this fee until the marginal increase in expected profit from the project is offset by this marginal consulting cost. The firm's best reply is, of course, also its inverse demand function for consulting. We therefore assume that

$$\frac{\partial^2 p_O}{\partial e_O \partial \tau} + \frac{\partial^2 p_O}{\partial e_O^2}c < 0 \tag{7}$$

to insure that this demand curve, and the firm's best reply function, are negatively sloped. Further note that, because effective funding $e_O$ also depends on funding for the researcher's university project, ($6a$) shows how spillovers from basic university research can influence the firm's unit consulting fee, and so whether our heroine actually consults.

However, devoting more time to consulting has two conflicting effects on the researcher's expected utility. First, for any fee, more time in consulting increases her income, whether either research project succeeds or not, as shown by the second term in ($6b$). However, the first term in ($6a$) shows that diverting more time to consulting also decreases her expected utility by decreasing the probability of success in university research, and thus the probability of the resulting reputational enhancement and license revenue. If the expected loss of utility from diverting any time to consulting is too high, then she will not do so. Otherwise, she increases time in consulting until the marginal gain in expected utility from consulting income is offset by this marginal expected loss in her university research.

Naturally our heroine does not consult for free. At $c = 0$, her expected marginal utility from time in consulting is negative, because diverting time from her university project decreases the probability of success in it, and thus her expected utility, without providing any additional income in return. Therefore, she never consults unless her expected marginal utility is increasing in the consulting fee $c$ for at least some values, so that as $c$ increases, her expected marginal utility, and her best reply, eventually become positive.

**Theorem 2** *The researcher's best reply function in consulting time, $\hat{t}(c)$, is positive only if $c_m = \min\{c : \frac{\partial EU(G,F,0,c)}{\partial t} = 0\}$ exists and is finite. If so, then $c_m > 0$, and her best reply is positive and increasing in the consulting fee, $\hat{t}(c) > 0$ and $\hat{t}'(c) > 0$, for all fees in a neighborhood above $c_m$.*

Even at positive fees, she does not divert time from her university research into consulting unless the certain gain from consulting income plus the expected gain from licensing income exceeds the expected loss of reputational enhancement. Thus, the slope of her best reply depends, in general, upon both her attitude toward risk and the trade-off in her preferences between income and reputation. For example, suppose her marginal utility of income increases when her university research succeeds, $\frac{\partial U(R_s,A+\gamma L+ct)}{\partial Y} > \frac{\partial U(R_f,A+ct)}{\partial Y}$. Then her best reply is negatively sloped whether she is risk-neutral or risk-averse. This is the case where, roughly speaking, income and reputation are complements in consumption ($\frac{\partial^2 U}{\partial Y \partial R} > 0$). Higher consulting fees and higher income for any given time spent consulting enhance the marginal value of additional reputation, which leads our heroine to devote more time to her university research.

Conversely, if her marginal utility of income does not increase when her university research succeeds, $\frac{\partial U(R_s,A+\gamma L+ct)}{\partial Y} \leq \frac{\partial U(R_f,A+ct)}{\partial Y}$, then her best reply is positively sloped if she is risk-neutral. In this case, her best reply may become negatively sloped, but only if she is risk-averse, and this risk aversion outweighs the "substitution effect" between income and reputation.

**Theorem 3** *When the researcher's best reply is interior, $\hat{t}(c) \in (0, M)$:*
*(i) If her marginal utility of income increases when her university research succeeds, $\frac{\partial U(R_s,A+\gamma L+ct)}{\partial Y} > \frac{\partial U(R_f,A+ct)}{\partial Y}$, then her best reply function for consulting time is decreasing in the consulting fee, whether she is risk-neutral or risk-averse.*
*(ii) If her marginal utility of income does not increase when her university research succeeds, $\frac{\partial U(R_s,A+\gamma L+ct)}{\partial Y} \leq \frac{\partial U(R_f,A+ct)}{\partial Y}$, then her best reply function for consulting time is:*
*(a) increasing if she is risk-neutral or not too risk-averse; and*
*(b) decreasing only if she is sufficiently risk-averse.*

Combining the results of Theorems 2 and 3, consulting occurs in equilibrium only if her best reply is initially increasing in the consulting fee. However, if she is risk-averse, then as the fee and her certain income increase, her best reply may eventually reach a maximum and become negatively sloped thereafter. Therefore, her best reply can be either positively or negatively sloped in equilibrium, as depicted in Figures 1 and 2.[13] This is not surprising, of course, because her best reply is also her consulting supply function.

In Section 2.2, we claimed that our approach allows us to focus on any given stage in the life cycle of this researcher, because the results depend on the stage of the life cycle only to the extent that they depend on the relative

[13]There is, of course, the possibility that her best reply not only intersects the firm's when it is increasing, but also turns down so sharply that it intersects the firm's again from above. In this case, however, the latter equilibrium is not locally stable, so we do not consider it.

magnitudes of her stocks of academic reputation and wealth. This follows from the results of Theorems 2 and 3, which show that the slope of our heroine's best reply (consulting supply) function, $\hat{t}(c)$, depends on the relative magnitudes of $\frac{\partial U(R_s, A+\gamma L+ct)}{\partial Y}$ and $\frac{\partial U(R_f, A+ct)}{\partial Y}$, and thus on how her marginal utility of income varies with the relative magnitudes of $R$ and $W$. Our results, therefore, depend on the stage of the life cycle only if the slope of her consulting supply varies over her life cycle, and if a change in this slope implies different results.

We therefore summarize the comparative statics of the consulting subgame equilibria in terms of the slopes of her consulting supply.

**Theorem 4** *In the equilibrium of the second stage consulting subgame:*
*(i) Independently of the slope of the researcher's best reply function:*
*(a) An increase in the extent $\beta$ to which her university research spills over into consulting, the research support $K_O$ provided by the firm in its lab, or the difficulty $x_O$ of the firm's project decreases the consulting fee, $\frac{\partial c^*}{\partial j} < 0$ for $j = \beta, K_O, x_O$.*
*(b) An increase in the fraction $\alpha$ of industrial funding that is equivalent to government funding, the research funding $K_I$ provided by the university, , license revenue $L$, or her share $\gamma$ of it decreases consulting time and increases the fee, $\frac{\partial t^*}{\partial j} < 0$ and $\frac{\partial c^*}{\partial j} > 0$ for $j = \alpha, K_I, L, \gamma$.*
*(c) In the special case of $\beta = 0$, an increase in government funding $G$ decreases consulting, $\frac{\partial t^*}{\partial G} < 0$.*
*(ii) If her best reply is positively sloped:*
*(a) An increase in $\beta$, $K_O$, or $x_O$ decreases consulting, $\frac{\partial t^*}{\partial j} < 0$ for $j = \beta, K_O, x_O$.*
*(b) An increase in $q$ has an ambiguous effect on consulting but increases the fee $\frac{\partial c^*}{\partial q} > 0$.*
*(c) An increase in $G$ or industrial funding $F$ decreases consulting, $\frac{\partial t^*}{\partial j} < 0$ for $j = G, F$, but has an ambiguous effect on the fee.*
*(d) An increase in her net assets $A$ increases consulting time and decreases the fee, $\frac{\partial t^*}{\partial j} > 0$ and $\frac{\partial c^*}{\partial j} < 0$.*
*(iii) If her reply is negatively sloped:*
*(a) An increase in $\beta$, $K_O$, or $x_O$ increases consulting, $\frac{\partial t^*}{\partial j} > 0$ for $j = \beta, K_O, x_O$.*
*(b) An increase in her net assets $A$ decreases consulting time and increases the fee, $\frac{\partial t^*}{\partial j} < 0$ and $\frac{\partial c^*}{\partial j} > 0$.*
*(c) If, in addition, her marginal utility of income does not increase if her university project succeeds, $\frac{\partial U(R_s, A+\gamma L+c^*t^*)}{\partial Y} \leq \frac{\partial U(R_f, A+c^*t^*)}{\partial Y}$, then*
*(1) An increase in $q$ decreases consulting and increases the fee, $\frac{\partial t^*}{\partial q} < 0$ and $\frac{\partial c^*}{\partial q} > 0$*
*(2) An increase in $G$ or $F$ must either decrease consulting, decrease the fee, or both (for $j = G, F$, either $\frac{\partial t^*}{\partial j} < 0$, $\frac{\partial c^*}{\partial j} < 0$, or both)*

These results are easily seen from Figures 1 and 2, which depict the cases where her best reply is positively and negatively sloped, respectively. First, an increase in $\beta$, $K_O$, or $x_O$ has no effect on the researcher's best reply, but the firm is willing to pay less per unit of time for her as a consultant, so its best reply shifts down. Consulting time $t^*$ decreases when her best reply is positively sloped, and increases when it is negatively sloped. In either case, the fee $c^*$ decreases. The effect of changes in these parameters may vary over the life cycle.

Conversely, an increase in $\alpha$, $K_I$, $\gamma$, or $L$ has no effect on the firm's best reply, but shifts hers left. She chooses to spend less time consulting for any fee, so $t^*$ decreases and $c^*$ increases whatever the slope of her consulting supply, and therefore at any stage of the life cycle.

Next, although an increase in $A$ also has no effect on the firm's best reply, it shifts her best reply to the right (left) if and only it is positively (negatively) sloped. That is, she chooses to consult more (less) for any given fee, so $t^*$ increases (decreases)and $c^*$ decreases (increases), if her consulting supply is positively (negatively) sloped. The effect of a change in $A$ also may vary over the life cycle.

With an increase in $q$, however, both best reply functions shift: the firm's shifts up because it is willing to pay more per unit of time, but hers shifts left because she is willing to consult less for any fee. In either case, $c^*$ increases. The change in $t^*$ is ambiguous if her best reply is positively sloped, but $t^*$ must decrease if it is negatively sloped. It is worth noting that this last result does not imply that higher quality researchers consult less, in general. Instead, it implies that, for any given consulting opportunity, a higher quality researcher will command a higher unit fee and spend less time consulting on that project. It is possible that higher quality researchers will have more consulting opportunities, but spend less time and earn a higher fee for each of them.

It is important to understand how changes in the levels of government and industrial funding chosen in stage one influence the stage-two consulting equilibrium. An increase in either $G$ or $F$ shifts the firm's best reply down, because it is willing to pay less for consulting, and shifts the researcher's best reply left, because she is willing to consult less. When her best reply is positively sloped, consulting time $t^*$ must decrease, but the effect on the fee $c^*$ is ambiguous, depending upon the relative magnitudes of these shifts. When her best reply is negatively sloped, the ultimate changes in both $c^*$ and $t^*$ are ambiguous. However, when the equilibrium is locally stable, as shown in Figure 2, then both equilibrium values cannot increase, or even remain constant. Either $c^*$ or $t^*$ must decrease.

Although changes in $\beta$ and $G$ have generally ambiguous results, we can state one result related to both that is unambiguous. As just noted, when $\beta > 0$, an increase in $G$ shifts the firm's best reply down and the researcher's best reply left, so the effect on $t^*$is uncertain. However, if $\beta = 0$, only the researcher's best reply shifts left if $G$ increases, so $t^*$decreases whatever the stage of the life cycle.

Finally, it is also worth noting that this result has important implications for the effects of the Bayh-Dole Act. Specifically, because this act gave rights from governmently funded patents to universities and their researcher-inventors, its passage was equivalent to increase in license revenue $L$ and the researcher's share of it $\gamma$. Our analysis shows that, whatever the slope of the researcher's best reply (consulting supply), and whatever the stage of the life cycle, passage of this act would tend to reduce the time spent by researchers in consulting and increase their consulting fees. That is, our model predicts that the potential for income from their own university research would lead them to substitute time in university research for consulting. This is important because many have expressed concern that this act could lead to less fundamental research. Our analysis, however, implies there is no reason to expect this effect, which is consistent with empirical studies that have failed to find such an effect (Azoulay *et al.* 2006, 2007; Thursby and Thursby 2007).

## 3.2 Stage One Equilibrium

In the first stage, the government funding agency and the firm simultaneously choose funding levels for the researcher's university project. As assumed above, the firm allocates a fixed amount $B_f > 0$ to R&D, and does not make major adjustments until the next budget cycle. Similarly, it is realistic to assume that the research budget of the government funding agency is also fixed at the level $B_g > 0$ during this period. To determine subgame perfect equilibria, we assume these funding choices are also made subject to equilibrium behavior in stage two, as detailed in the preceding subsection and embedded in the equilibrium functions $t^*(G, F)$ and $c^*(G, F)$. Substituting these into (3) and (4) gives the "reduced form" payoffs

$$P_g(G, F) = EU_g(G, F, t^*(G, F), c^*(G, F)) \tag{8}$$

and

$$P_f(G, F) = E\Pi(G, F, t^*(G, F), c^*(G, F)). \tag{9}$$

By construction, a Nash equilibrium $(G^*, F^*)$ of the simultaneous-move game with these payoffs is a subgame perfect equilibrium of the two-stage funding game.

**Theorem 5** *Consider the strategic form game with the government funding agency and firm as the players, whose strategies are $G \in [0, B_g]$ and $F \in [0, B_f]$, and payoff functions are defined by (8) and (9). Also assume each player's payoff function is continuous and strictly quasi-concave in its own strategy, given any strategy choices by the other players. Then this game has a Nash equilibrium $(G^*, F^*)$, and $(G^*, F^*, t^*(G^*, F^*), c^*(G^*, F^*))$ is the subgame perfect equilibrium of the two-stage funding game.*[14]

Maximization of (8) by choosing $G \in [0, B_g]$ implicitly defines a best reply function $\hat{G}(F)$, giving the level of government funding for university research that maximizes the agency's expected utility for any choice of funding $F$ by the firm. Similarly, maximization of (9) by choosing $F \in [0, B_f]$ implicitly defines a best reply function $\hat{F}(G)$, giving the level of industrial funding for university

[14] Again, these equilibrium values are also functions of all the parameters of the model ($\alpha$,$\beta$,$q$,$x_I$,$K_I$,$x_O$,$K_O$,$S$,$L$, $\gamma$).

research that maximizes the firm's expected profit for any funding level chosen by the government agency.[15] Again, however, because we are interested in deriving testable implications, we focus on the interior equilibrium of this funding game.

If the Nash equilibrium is interior, $G^* \in (0, B_g)$ and $F^* \in (0, B_f)$, then it must satisfy

$$\frac{\partial P_g(G^*, F^*)}{\partial G} = 0, \tag{10a}$$

and

$$\frac{\partial P_f(G^*, F^*)}{\partial F} = 0 \tag{10b}$$

where

$$\frac{\partial P_g(G, F)}{\partial G} = \left(\frac{\partial p_I}{\partial e_I} - \frac{\partial p_I}{\partial \tau}\frac{\partial t^*}{\partial G}\right)[U_g(R_{gs}) - U_g(R_{gf})] - V'(G) \tag{11a}$$

and

$$\frac{\partial P_f(G, F)}{\partial F} = \left(\frac{\partial p_I}{\partial e_I}\alpha - \frac{\partial p_I}{\partial \tau}\frac{\partial t^*}{\partial F}\right)(\pi_I - L) + \left(\frac{\partial p_O}{\partial \tau}\frac{\partial t^*}{\partial F}\right)\pi. \tag{11b}$$

As expected, these conditions show both the initial marginal trade-offs between the benefits and costs of funding, and the effects of initial funding choices on the second stage equilibrium values. Notice the effect of these choices on the equilibrium consulting fee $c^*$ does not directly enter the decision of either the firm or the government funding agency. The agency's payoff does not depend on $c^*$, and firm's second stage optimal choice of $c^*$ eliminates its effect on the first stage funding choice (via a standard envelope theorem application). An example of this equilibrium is depicted in Figure 3.

The conditions in ($10a$) and ($11a$) essentially show that increases in government funding directly increase effective funding $e_I$, and thus both the probability of success and expected utility, so the agency increases $G$ until this marginal increase in expected utility from this project is offset by the marginal cost of reduced funding to other projects (embedded in $V$). Note that $\frac{\partial p_I}{\partial e_I} - \frac{\partial p_I}{\partial \tau}\frac{\partial t^*}{\partial G} > 0$ if $\frac{\partial t^*}{\partial G} < 0$, in which case it follows from ($11a$) that the agency's best reply is interior as long as the opportunity cost of funding our heroine is not too high. The conditions in ($10b$) and ($11b$) show that devoting more funds to our heroines's university research has conflicting effects for the firm. First, it increases the probability of success in university research, and expected licensing profit. However, if $\frac{\partial t^*}{\partial F} < 0$, this reduces time in consulting, and therefore the probability of success in and expected profit from the firm's project. In this case, the firm funds university research as long as the increase in expected profit from licensing a success from the university project outweighs the expected profit loss from its own project.

[15] We omit the parameters of the model as explicit arguments in these best reply functions for clarity of exposition.

Given the general ambiguity of $\frac{\partial t^*}{\partial G}$ and $\frac{\partial t^*}{\partial F}$, it is difficult to obtain comparative statics results on equilibrium levels of funding for university research because $\frac{\partial t^*}{\partial G}$ and $\frac{\partial t^*}{\partial F}$ are negative when the researcher's best reply is positively sloped but at least one of them must be negative when this best is negatively sloped. Indeed, even the slopes of the government and firm best reply functions are not obvious. Nevertheless, we can obtain results under reasonable assumptions. To do so we assume that

$$-\frac{\partial^2 p_I}{\partial \tau^2}\frac{\partial t^*}{\partial G} + \frac{\partial^2 p_I}{\partial \tau \partial e_I} > 0, \tag{12a}$$

$$-\frac{\partial^2 p_I}{\partial \tau^2}\frac{\partial t^*}{\partial F} + \alpha\frac{\partial^2 p_I}{\partial \tau \partial e_I} > 0, \tag{12b}$$

$$\frac{\partial^2 p_I}{\partial e_I^2} - \frac{\partial^2 p_I}{\partial \tau \partial e_I}\frac{\partial t^*}{\partial G} > 0, \tag{12c}$$

and

$$\alpha\frac{\partial^2 p_I}{\partial e_I^2} - \frac{\partial^2 p_I}{\partial \tau \partial e_I}\frac{\partial t^*}{\partial F} > 0. \tag{12d}$$

These conditions essentially state that a stage-one increase in one type of external funding increases the marginal effect of the other type of external funding on the stage-two equilibrium probability of success in university research. That is, $(12a, c)$ imply that an increase in $F$ increases the marginal effect of $G$ on $p_I(T - t^*, e_I; q, x_I)$, and $(12b, d)$ imply that an increase in $G$ increases the marginal effect of $F$ on $p_I(T - t^*, e_I; q, x_I)$. Under these assumptions we have the following.

**Theorem 6** *Assume that equilibrium consulting time is decreasing in firm funding,* $\frac{\partial t^*}{\partial F} < 0$, *and, that* (12) *holds, and second-order effects on equilibrium consulting times are negligible,* $\frac{\partial^2 t^*}{\partial i \partial j} \approx 0$ *for all parameters* $i$ *and* $j$. *Then:*
*(i) The first-stage best reply function of the funding agency is positively sloped.*
*(ii) The first-stage best reply function of the firm is positively sloped if, in addition, an increase in government funding decreases equilibrium consulting time,* $\frac{\partial t^*}{\partial G} < 0$ *and sufficiently decreases equilibrium effective funding for the firm's consulting project,* $\frac{\partial e_O^*}{\partial G} \leq -(\frac{\partial^2 p_O}{\partial \tau^2})(\frac{\partial t^*}{\partial G})/\frac{\partial^2 p_O}{\partial \tau \partial e_O}$.

Two points should be noted. First, the conditions in (12) and a negative effect of firm funding on equilibrium time spent consulting are sufficient for the government's best reply function to be positively sloped. Second, the additional hypotheses in (ii) of this theorem states that, although there are spillovers from university research to the firm's project, they cannot outweigh the reduced time the researcher spends in consulting if $G$ increases, so equilibrium effective funding $e_O^*$ for the firm's project decreases. This guarantees that an increase in $G$ also results in a decrease in the marginal effect of $F$ on the stage-two equilibrium probability of success in the firm's consulting project.

When the best reply functions are positively sloped, as depicted in Figures 3 and 4, we can identify some comparative statics results for the first stage.

**Theorem 7** *Under the hypotheses of Theorem 6, if the first-stage equilibrium is locally stable, then an increase in research funding within the university, license revenue, or her share of it must increase equilibrium university research funding from both the government and the firm,* ($\frac{\partial G^*}{\partial j} > 0$ *and* $\frac{\partial F^*}{\partial j} > 0$ *for* $j = K_I, L, \gamma$) *if, in addition, this sufficiently decreases equilibrium effective funding for the firm's consulting project,* $\frac{\partial e_O^*}{\partial j} \leq -(\frac{\partial^2 p_O}{\partial \tau^2})(\frac{\partial t^*}{\partial j})/\frac{\partial^2 p_O}{\partial \tau \partial e_O}$ *for* $j = K_I, L, \gamma$.

An increase in the level of university research support, license revenue, or her share of it shifts the agency's best reply up which must increase funding for her university research from both external sources. In each of these cases, the additional hypothesis is sufficient, but not necessary, to guarantee that the firm's best reply shifts right. Because each of these changes has an ambiguous effect on total consulting expenditure $c^*t^*$, the condition $\frac{\partial e_O^*}{\partial j} \leq -(\frac{\partial^2 p_O}{\partial \tau^2})(\frac{\partial t^*}{\partial j})/\frac{\partial^2 p_O}{\partial \tau \partial e_O}$ states that this effect is either negative, or not too positive. All other changes are ambiguous in this case. Nevertheless, we can show the following limited results.

**Corollary 8** *If the effects of first stage changes on second stage equilibrium values are sufficiently small, and the equilibrium is locally stable, then:*
*(i) An increase in the extent to which her university research spills over into consulting, the research support provided by the firm in its lab, or the difficulty of the firm's research project must decrease equilibrium university research funding from both the government agency, decrease equilibrium university research funding from the firm, or both* ($\frac{\partial G^*}{\partial j} < 0$ *and* $\frac{\partial F^*}{\partial j} < 0$ $j = \beta, K_O, x_O$).
*(ii) An increase in the quality of the researcher must increase equilibrium university research funding from both the government agency and the firm* ($\frac{\partial G^*}{\partial q} > 0$ *and* $\frac{\partial F^*}{\partial q} > 0$).

We emphasize caution in interpreting these results, because they are derived by minimizing the effects of parametric changes on the second stage consulting equilibrium. Nevertheless, we do find this instructive, because it focuses on the "short-run" effects of changes on the marginal probability of success in the university research project. Because research quality and effective funding are "complements" in production, $\frac{\partial^2 p_I}{\partial e_I \partial q} > 0$, higher quality researchers are more likely to receive higher levels of external funding from either the government agency or industry, ceteris paribus. That is, both best reply functions shift outward, so the researcher definitely receives more funding. The ambiguity in the general case results from the fact that changes in quality and external funding have conflicting effects on time spent in consulting. Also in this case, increases in $\beta$, $K_O$, or $x_O$ have no effect on the government agent's choice, and the researcher definitely receives less external funding.

## 3.3 Extensions

The game structure used above conforms well to the stylized fact that faculty typically prefer their own research, and therefore focus obtaining funds for it before making any consulting agreement. Similarly, faculty also prefer government funding to industrial funding because the former has fewer ties on its use. We explicitly incorporated this notion in the definition of effective funding for university research. Thus, one might wonder whether it is more reasonable to consider a game structure in which our heroine seeks support from the government agency first, then (possibly) seeks support from the firm after learning how much the agency provides, followed again by the consulting game. In this subsection we consider this sequence of events diagrammatically.

Because there is no change in the final stage, we can focus on the first stage. The government agency acts as a Stackelberg leader, choosing the point on the firm's reaction function that gives it the greatest expected utility. Because funding levels are strategic complements, the agency takes advantage of its leadership position to provide more funding (than when they move simultaneously), and so induce the firm to provide more funding. That is, the equilibrium in this game is a point on the firm's best reply "northeast" of where it intersects the government's best reply. Comparative statics results for this stage essentially follow from Theorems 6 and 7, because parametric changes shift the best replies as before, and we know the new equilibrium for this game will be northeast of the new simultaneous-move equilibrium point on the firm's best reply. For example, an increase in either $\gamma$, $L$, or $q$, which increases both best replies, must result in higher firm funding $F^*$, though government funding $G^*$ may increase or decrease. Similarly, an increase in either $\beta$, $K_O$, or $x_I$, must result in lower firm funding, though government funding may increase or decrease.

# 4 Econometric Analysis

In this Section we focus on empirical estimates of the two stages of the model. While the ideal data would include time spent in consulting and the fee paid, such data are not available to our knowledge. However, because our theoretical focus in on consulting that is essentially research in firm labs, we are able to exploit a unique data set of 1690 patents on which 458 faculty from eight major US universities are listed as inventors. Thirty percent of these patents are assigned to (and therefore owned by) firms. From interviews with faculty, university licensing professionals, and firm R&D executives (Thursby et al. 2007a) the point was made that patents assigned to firms are typically the outcome of consulting. Thus our measure of faculty consulting is based on assignment pattern.

Our analysis considers the of faculty at Purdue, MIT, Stanford, Wisconsin, Georgia Tech, Cornell, Pennsylvania and Texas A&M. For each of the faculty at these universities we have detailed annual information on faculty publications, citations and research funding. We restrict attention only to faculty in years in which they have applied for a patent which is granted between 1993 and 1999. This yields 1690 patent/inventor pairs where assignment of the patent is either to the university or to a firm.[16] These pairs include 1532 patents and 458

[16] A number of the firms in the sample are firms in which the inventor is a principal (founder,

faculty inventors. In our econometric analysis of assignment we randomly drop duplicate patents so that we consider each patent once.

While we have detailed information for all faculty for all years at the eight universities, we use only the information for years in which a faculty member is known to have applied for a patent. Our reason is simple. The model we develop is for faculty who can conceivably consult with industry. If a university inventor applies for a subsequently granted patent in some year then clearly some of their work was deemed to be useful and hence one can argue that they could have consulted in that year even if they did not consult. Moreover, since our only measure of consulting is assignment of a patent to a firm, then for reasons of comparability across the sample we want also to restrict non-consulting observations to years in which a university assigned patent is applied for. Finally, we use patent characteristics as a measure of the focus of research. By restricting attention only to patent application years we miss faculty who could or might have consulted with industry; more importantly, however, we exclude faculty who were not of interest to industry as consultants.

It is also important to recognize that while the theoretical model yields a number of hypotheses that are testable in principle, a number of these concern the consulting fee $c^*$ which we do not observe. Recall also that a number of results depend on the slope of the researcher's consulting supply function or the slopes of the funding best reply functions, and hence are not testable. Thus much of what follows is properly regarded as estimation of the system, rather than testing.

## 4.1 Consulting

For the second stage regression we use a logit regression to explain the probability that a patent is assigned to a university, $P(UNIVASSGN_i = 1)$, rather than to a firm, $P(UNIVASSGN_i = 0)$,where $i$ refers to a patent/inventor pair. Since assignment of a faculty patent to a firm is largely the outgrowth of consulting we interpret the probability of assignment as a measure of time spent in consulting $t^*$.

According to our model regressors in the logit model should include measures of government and industry funding, $G$ and $F$, respectively, the researcher's quality, $q$, the difficulty (or scientific merit) of both the university and firm projects, $x_I$ and $x_O$, research support provided by the university and firm, $K_I$ and $K_0$, the inventor's share $\gamma$ of university license revenue $L$, the fraction alpha of $F$ that is equivalent to $G$, and the extent to which the university research contributes to the firm's problem $\beta$. For these variables we have direct measures only for $G$, $F$, $q$, and $\gamma$.[17]

CEO and/or member of the scientific advisory board). However, our model does not differentiate between consulting with a start-up and consulting with an established firm thus our empirical analysis does not differentiate patents assigned to start-ups from other firm assigned patents.

Also an additional 80 patents were found but these were either unassigned or they had multiple assignees.

[17] The aggregate annual amount of licensing revenue by university is available. However,

While the comparative statics for the consulting stage yield a number of testable hypotheses, many of those pertain to $c^*$ for which we have no data. The two hypotheses which we can test are (i) an increase in $\gamma$ should decrease $t^*$, and (ii) if there are no spillovers, i.e., $\beta = 0$, then an increase in government funding should decrease $t^*$.

For each faculty member we include yearly total US government sponsored research funds and the yearly total industry sponsored research funds received in the year prior to the patent application ($LAG\_GOV\_FND$ and $LAG\_IND\_FND$) as our measures of $G$ and $F$. For those universities with sliding scales for $\gamma$, the inventor's share of university licensing income, we use the inventor share ($INVENTSHARE$) for income between \$25k and \$50k since the average licensing revenue for an active license in the US lies between those figures (AUTM, various years). University fixed effects should provide a control for $K_I$, but we do not have a proxy for $K_0$. When university fixed effects are used we cannot also include $INVENTSHARE$.

To control for $x_I$ and $x_O$ we use several measures of patent characteristics. Three of the measures are backward looking. The first is the number of backward citations to prior patents ($PATENT\_CITES$) contained in the focal patent. The larger the number of backward citations the larger is the existing body of related patented work, so that we would expect patents with more backward citations to be more incremental and hence of less scientific merit. The second backward looking measure is the Trajtenberg et al. (1997) measure of patent originality ($ORIGINAL$). $ORIGINAL$ is based on a Herfindahl index that reflects the dispersion of citations made by the patent across patent classes. The originality score is higher the wider the range of classes to which the patent makes citations. A score of zero indicates that all citations to prior art are in a single patent class and scores close to one indicate citations to many classes. A patent is considered more original if it cites prior art from many rather than few technology classes. Both $PATENT\_CITES$ and $ORIGINAL$ are from the NBER Patent Database (Hall et al., 2001). We also include as an additional backward measure the number of non-patent publications cited as prior art in the patent ($ARTICLE\_CITES$). As a forward looking measure we include the number of forward citations ($FOR\_CITES$) received by the patent by October 2006. It reflects importance of the patent in the sense that the patent has been considered prior art by either subsequent inventors or patent examiners.

For researcher quality we use the number of publications by the faculty member in the year prior to the patent application ($LAG\_PUBS$) and the total number of citations those publications received through 2003 ($LAG\_PUB\_CITES$). While this latter variable may signal inventor quality, it is likely also that faculty who conduct more fundamental work are cited more (holding constant the number of publications). Thus $LAG\_PUB\_CITES$ may reflect both inventor quality $q$ and $x_I$.

Additional controls are indicator variables for major program field of the in-

the appropriate value of $L$ is the licensing revenue that would accrue to the university project if it is licensed.

ventor: $PHYSCI = 1$ if the inventor is in the physical sciences and $ENG = 1$ for engineering faculty; the excluded category is biological sciences. When university fixed effects are not included we include an indicator variable for public versus private university ($PUBLIC = 1$ if the university is public) and an indicator variable for whether the university is located in an urban area ($URBAN = 1$ if the university is located in an urban area). Thursby et al. (2007a) suggest that urban areas might provide more opportunities for consulting. Public universities often are expected to interact with (particularly local) firms to meet economic development goals (Thursby et al., 2007a, Sharon Belonzon and Mark Schankerman, 2007).

Final controls are included for the age of the inventor at the time of the patent assignment ($AGE$) and the inventor's gender ($MALE = 1$ if the inventor is male). Thursby and Thursby (2007) find significant gender differences in faculty propensity to engage in licensing activities and Azoulay et al. (2007) find significant gender effects on faculty patent activity. Thursby et al. (2007b) argue that age effects on faculty commercialization activities are non-linear thus we also include the square of age ($AGESQ$). In our model we show a relationship between inventor assets and consulting. To the extent that assets rise with age, we expect age and assets to be positively related.

Summary statistics are found in Table 1 and the logit results in terms of odds ratios are given in Table 2. Note that we use logs of $LAG\_GOVFND$, $LAG\_INDFND$, $LAG\_PUBS$, $LAG\_PUB\_CITES$, $PATENT\_CITES$, $ARTICLE\_CITES$ and $FOR\_CITES$ since these variables are skewed. Robust standard errors are used. In the Part a $PUBLIC$, $INVENTSHARE$ and $URBAN$ are included while in part b those variables are dropped so that a university fixed effects model can be estimated.

In each specification the higher quality faculty, as measured by publications, consult less. Citations to publications, however, have a negative but insignificant effect on consulting. Since citations can also be interpreted as a measure of $x_I$, the difference in results suggests that $LAG\_PUB\_CITES$ is picking up both quality and the nature of the inventor's university research. It is also the case that faculty with greater government funding are more likely to consult. Greater industrial funding is associated with a higher probability of assignment to the university.

Recall from our theoretical analysis, the impact of government funding on consulting should depend on the existence of spillovers. When $\beta > 0$, an increase in government funding shifts the researcher's best reply back and the firm's down, hence the ambiguous theoretical results. When $\beta = 0$, only the researcher's best reply shifts with an increase in government funding, implying a decrease in consulting. The positive empirical relationship between government funding and consulting is possible only if $\beta > 0$, thus the presence of a spillover is accepted by the data. Further, recall from Theorem 4 that with a spillover, an increase in funding will increase consulting only when the resaercher's best reply function is negatively sloped in equilibrium. With a negatively sloped function, the theory also predicts the negative impact of quality on consulting that we find empirically.[18]

From the theory, $INVENTSHARE$ should decrease consulting regardless of the researcher's best reply. We find that it has the correct sign but it is not significant at conventional levels. This lack of precision in our estimates of the coefficient of the inventor's share is not surprising since there is little variation in the shares across the 8 universities. Five of the universities provide 33% as the share and the other three provide 20%, 30% and 52%. However, when we drop $MALE$, which has a very small t-statistic in both regressions, the coefficient of $INVENTSHARE$ is positive in accordance with our model, though it is significant only at a 10% level.

In both specifications, the coefficients of $AGE$ and $AGESQ$ are individually and jointly significantly different from zero. In both regressions the marginal effect of another year is positive until around age 54, at which point the marginal effect of age becomes negative.

The patent characteristic variables are consistent with our assumption that $x_I > x_0$. Specifically, the measure of patent originality ($ORIGINAL$) is associated with a higher probability of assignment to the university and it is significant at the 1% level. The larger the number of backward patent citations ($PATENT\_CITES$) the greater is the likelihood that the patent is assigned to a firm, thus the more incremental patents are assigned to the firm. $FOR\_CITES$ is not significant in either regression. Finally, the more articles cited (as opposed to citations to prior patents), the more likely it is that the patent is assigned to the university. This is the opposite of the effect of backward patent citations, and, while it might contradict the claim that firm patents are more incremental, it is likely only a sign that university inventions are closer to the academic literature than are firm inventions.

A number of robustness checks were considered. In the first we include the "expected" number of citations for a researcher's publications. This is computed as the average number of citations received by articles in the journals where the researcher's publications appear. Expected citations are not significant and other results are unchanged. When all variables are entered linearly rather than in logarithms results change little. The two age variables are now not significantly different from zero. In the regression with fixed effects, $LAG\_PUB\_CITES$ now has a negative effect on consulting and it is significant at the 5% level. The lags of publications, publication citations, and government and industry funding are used based on our prior that the lagged measures are the appropriate measures of effects on consulting. When we use the current year values the only change of note is the now insignificance of government funding in the model with fixed effects.

[18] It is tempting to argue that federal funding is another sign of inventor quality. Since the peer review processes followed by federal agencies identifies, to some extent, the best researchers in a given field of inquiry then those with the larger amounts of federal funding are, in general, higher quality researchers. However, the effect of additional federal funding is opposite that of publications which are clearly measures of researcher quality.

## 4.2 Government and Industry Funding

The funding stage regressions explain both the amount of government research funding, $GOV\_FND$, and industry research funds, $IND\_FND$, received by an inventor in a year in which they applied for a patent. In our model, each type of funding depends on the other. We also include the lagged value of the dependent variable.

As in the consulting stage, we use $LAG\_PUBS$ and $LAG\_PUB\_CITES$ as measures of inventor quality. Lagged rather than current publications and citations are used to allow for a lag between funding applications and their funding. Lagged publications and citations most likely reflect the researcher's productivity at the time the funding was applied for. While it is standard to consider citations as a measure of quality, it is also likely to be the case, as we noted above, that more highly cited faculty conduct more fundamental research. Also, as in the consulting stage, when university fixed effects are not included we include the percentage of licensing revenue the university awards the inventor ($INVENTSHARE$).

Under the conditions of Corollary 8, an increase in $\beta$ decreases the equilibrium levels of government and firm funding. Our measure of the spillover, $SPILL$, is the number of article citations in the patent to the inventor's prior research. That is, it is a count of the number of articles authored by the inventor that are cited as prior art in the patent. If we assume that the researcher's journal publications result primarily from solving her university problem, then the larger is $SPILL$ the more the patent relies on the inventor's university research. If the patent is assigned to a university then $SPILL$ is zero.[19] We view this as a measure of the spillover from the inventor's university research. Given that many of the inventors in our sample have multiple patents in a year we randomly select one of the patents to measure the level of spillover.[20]

To control for funding differences across fields we include indicator variables for the major program area of the inventor ($PHYSCI$ and $ENG$). Funding is also likely to vary according to the size of the lab in which the inventor works. While lab size may be a function of field, it might also be a function of the type of scientific issues addressed by the inventor. Unfortunately, we do not have the number (or composition) of the inventor's lab. However, it is common for scientific articles to include the names of most, if not all, of the members of a lab so that the number of an inventor's co-authors should be positively correlated with lab size. We use the average number of co-authors as a control for lab size, $LAB\_SIZE$, over a three year window that includes the year of the patent application, the year before the application and the year after the application. A three year average is used since the number of publications can vary substantially from year to year.

To account for the many zero dependent variable observations we use Tobit models. Since our theoretical model assumes that government and industry

[19] We cannot use this measure of spillover in the consulting regression since SPILL is always zero when UNIVASSGN=1 (that is, when the patent is assigned to a university).

[20] We are currently working on extending our measure to include all patents in a given year

sponsored research are simultaneously determined we use an instrumental variables estimator. Instruments for the endogenous funding levels are their lagged values. Results are found in Tables 3 and 4. In Part a of each table are results without university fixed effects while fixed effects are used in Parts b.

In Part b of Table 3, the university fixed effects specification, industrial funding is positively and significantly related to government funding. When fixed effects are included the coefficient of industrial funding is positive but it is not significantly different from zero. In both industry funding specifications in Table 4 government funding is positive and significantly different from zero at a 10% level or lower. These results are consistent with the result in Theorem 6 that government and firm funding act as strategic complements.

Corollary 8 gave sufficient conditions for $\beta$ to have a negative effect on government and industry funding. In the industry funding equation, Table 4, the effect is negative and significantly different from zero. However, $SPILL$ is not significantly related to government funding. Lastly, we do not find a significant relationship between $INVENTSHARE$ and either type of funding though it does have the predicted negative sign.

When we drop the lagged values from the equations we find several changes. Citations to publications are now positively and significantly related to government funding. Male inventors have significantly higher government funding than do females. $AGE$ and $AGESQ$ are now significantly different from zero in both government specifications. In the industry funding equations higher publications are associated with greater industry funding, but the opposite holds for citations to those publications.

# 5 Concluding Remarks

Despite survey results showing that industrial managers often consider consulting to be one of the more important mechanisms for industry to access university research, there is little research either theoretically or empirically of this mechanism. In this paper, we develop and estimate a model of university industry knowledge flows in the context of faculty consulting. Our theoretical model of consulting incorporates faculty decisions to conduct research within the university or outside in a firm's lab as well as the decisions of funding agents, both government and industrial, on support for the researcher's work within the university. The model yields predictions for the time spent consulting, the associated fee, and the level of government and industry support for university research as functions of faculty quality, project characteristics, the researcher's share of license revenue from the university project, R&D spillovers, university support for the researcher's internal project, as well as the willingness of the firm and government to sponsor the faculty member's research within the university.

In the consulting stage, we find that increases in the researcher's quality, university support for the researcher's internal project, or the researcher's share of license revenue from her university research lead to a greater consulting fee. By contrast, increases in the restrictions the firm places on university research funding, R&D spillovers, or the scientific merit of the firm's project all lead to a lower fee. We also find that an increase in restrictions placed by the firm on its university funding increases the amount of time spent consulting, while an increase in the researcher's share of license revenue decreases the time spent consulting.

The license share result is of particular note, since in this model a decrease in consulting implies an increase in time devoted to university research. If the university project is more basic than the firm's, then contrary to the policy concern that licensing might reduce basic research, the increase in share increases the time devoted to basic research.

In general the effects of government and firm funding on time spent consulting are ambiguous, nonetheless in the absence of R&D spillovers, an increase in government funding reduces the time spent consulting regardless of the slope. In the funding stage, we provide sufficient conditions for government and firm funding to act as strategic complements. Under additional plausible conditions, we find that the equilibrium levels of funding are positively related to quality and negatively related to spillovers.

The empirical results generally support the theory. Results for the consulting stage support our assumption that university research projects are more basic than firm projects. We also find that in the funding stage, government and industrial funding are strategic complements. Perhaps the most striking results are those regarding spillovers. In the consulting stage, we find that consulting is positively associated with government funding. In the context of our theoretical model, this result is possible only if there is a spillover from the faculty researcher's government sponsored research to the firm's research problem. As predicted by the theoretical results when government and firm research are strategic complements, we find that industry sponsored research in the funding game is negatively associated with a measure of spillovers that relates firm assigned faculty patents to the faculty member's academic publications.

Our combined theoretical and empirical results provide new insights into the ways in which firms benefit from spillovers from government funding for university research. In particular, our empirical approach of identifying faculty contributions to industrial patenting according to firm assigned patents with faculty inventors shows that spillovers are greater than those identified by the common practice of examining citations in firm assigned patents to university assigned patents. By explicitly modeling consulting as the mechanism involved, we are able to link these spillovers to the levels of research funding.

Several qualifiers to our work suggest directions for further research. First, one fourth of the patents in the sample assigned to for-profit firms are assignments to firms in which the inventor is a principal (founder, CEO, and/or scientific advisor). A role as scientific advisor is consistent with our interpretation of the faculty researcher choosing $t > 0$ and is consistent with most university policies as long at $t \leq M$. The patent may or may not be a follow-on patent to one from the faculty researcher's university research, in which case we would interpret the follow on project as $x_o$. Moreover, most conflict of interest policies prohibit faculty from receiving sponsored research from their start ups, so that this example would be the special case of our model in which $F = 0$. Of course, we do not differentiate between start ups and other types of firms in the analysis so we abstract from many of the nuances of faculty start ups.

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# 7 Appendix

I. Proof of Theorem 1.

Because the number of players is finite, their strategy sets are compact and nonempty, and their payoff functions are continuous and strictly quasi-concave, this follows directly from the well-known existence theorem for strategic form games with continuous strategy spaces (see, for example, Friedman 1977).

II. Proof of Theorem 2.

First observe from (6a) that $\frac{\partial EU(G,F,t,0)}{\partial t} = -\frac{\partial p_I}{\partial \tau}[U(R_s, A+\gamma L) - U(R_f, A)] < 0$ for all $t$ because $R_s > R_f$, $\gamma L > 0$, and positive marginal utility imply that $U(R_s, A+\gamma L) > U(R_f, A)$, and $\frac{\partial p_I}{\partial \tau} > 0$. Hence, because $t$ is constrained to be nonnegative, $\hat{t}(0) = 0$. That is, if we plotted $\frac{\partial EU(G,F,t,c)}{\partial t}$ as a function of $c$ for fixed $(G, F, t)$, then it would intersect the (vertical) utility axis at a negative value. Because $\frac{\partial^2 EU(G,F,t,c)}{\partial t^2} < 0$, if $\frac{\partial EU(G,F,0,c)}{\partial t} < 0$ for all $c \in [0, B_f/M]$, then $\frac{\partial EU(G,F,t,c)}{\partial t} < 0$ for all $c$, and consulting never occurs, $\hat{t}(c) = 0$ for all $c$. However, the slope of $\frac{\partial EU}{\partial t}$ with respect to $c$ at $c = 0$ is $\frac{\partial^2 EU(G,F,t,0)}{\partial t \partial c} = (-\frac{\partial p_I}{\partial \tau})[\frac{\partial U(R_s,A+\gamma L)}{\partial Y} - \frac{\partial U(R_f,A)}{\partial Y}]t > 0$ if $\frac{\partial U(R_s,A+\gamma L)}{\partial Y} < \frac{\partial U(R_f,A)}{\partial Y}$. Thus, it is possible that $\frac{\partial EU(G,F,t,c)}{\partial t}$ increases (though perhaps not monotonically) as $c$ increases, and eventually intersects the (horizontal) $c$ axis. If so, there exists a positive, finite $c_m$ defined as above. By continuity, $\frac{\partial^2 EU(G,F,0,c_m)}{\partial t \partial c} > 0$. Note that $c_m$ is the fee at which the function $EU(G, F, t, c)$ takes on its unconstrained maximum at $t = 0$ (or the smallest fee if this occurs for more than one value). Therefore, for all fees in a neighborhood above $c_m$, $EU(G, F, t, c)$ takes on its unconstrained maximum at some $t > 0$, so $\hat{t}(c) > 0$. Moreover, $\hat{t}'(c) > 0$ in this neighborhood from the proof of Theorem 2 because $\frac{\partial^2 EU(G,F,0,c_m)}{\partial t \partial c} > 0$.

III. Proof of Theorem 3

When her best reply is interior, its slope is $\frac{\partial \hat{t}(c)}{\partial c} = -(\frac{\partial^2 EU}{\partial t \partial c})/(\frac{\partial^2 EU}{\partial t^2})$, which has the sign of

$$\begin{aligned}\frac{\partial^2 EU}{\partial t \partial c} &= (-\frac{\partial p_I}{\partial \tau})[\frac{\partial U(R_s, A+\gamma L+ct)}{\partial Y} - \frac{\partial U(R_f, A+ct)}{\partial Y}]t + \\ &\quad [p_I \frac{\partial^2 U(R_s, A+\gamma L+ct)}{\partial Y^2} + (1-p_I)\frac{\partial^2 U(R_f, A+ct)}{\partial Y^2}]ct,\end{aligned} \tag{13}$$

because expected utility is assumed strictly concave in $t$. Hence, if she is risk-neutral, so $\frac{\partial^2 U}{\partial Y^2} = 0$, then because $\frac{\partial p_I}{\partial \tau} > 0$, the sign of $\frac{\partial^2 EU}{\partial t \partial c}$ is given by the sign of $-[\frac{\partial U(R_s,A+\gamma L+ct)}{\partial Y} - \frac{\partial U(R_f,A+ct)}{\partial Y}]$. Statement (i) follows immediately.

Statement (ii) then follows from $\frac{\partial p_I}{\partial \tau} > 0$ and the fact that if she is risk-averse, then $\frac{\partial^2 U}{\partial Y^2} < 0$, and the second term in (8) is negative.

IV. Proof of Theorem 4

Using standard comparative statics, $\frac{\partial t^*}{\partial j} = [\frac{\partial^2 EU}{\partial t \partial c}\frac{\partial^2 E\Pi}{\partial c \partial j} - \frac{\partial^2 E\Pi}{\partial c^2}\frac{\partial^2 EU}{\partial t \partial j}]/D_2$ and $\frac{\partial c^*}{\partial j} = [\frac{\partial^2 E\Pi}{\partial c \partial t}\frac{\partial^2 EU}{\partial t \partial j} - \frac{\partial^2 EU}{\partial t^2}\frac{\partial^2 E\Pi}{\partial c \partial j}]/D_2$, for $j = G,F,\alpha,\beta,q,x_I,\ K_I,x_O,K_O,A,L,\gamma$ where $D_2 = \frac{\partial^2 EU}{\partial t^2}\frac{\partial^2 E\Pi}{\partial c^2} - \frac{\partial^2 EU}{\partial t \partial c}\frac{\partial^2 E\Pi}{\partial c \partial t} > 0$ by the assumption that the equilibrium is locally stable. Differentiation yields $\frac{\partial^2 EU}{\partial t \partial G} = (-\frac{\partial^2 p_I}{\partial \tau \partial e_I})[U(R_s, A+\gamma L+ct) - U(R_f, A+ct)] + \frac{\partial p_I}{\partial e_I}[\frac{\partial U(R_s,A+\gamma L+ct)}{\partial Y} - \frac{\partial U(R_f,A+ct)}{\partial Y}]c$, $\frac{\partial^2 EU}{\partial t \partial F} = \alpha\frac{\partial^2 EU}{\partial t \partial G}$, $\frac{\partial^2 EU}{\partial t \partial \alpha} = F\frac{\partial^2 EU}{\partial t \partial G}$, $\frac{\partial^2 EU}{\partial t \partial K_I} = \frac{\partial^2 EU}{\partial t \partial G}$, $\frac{\partial^2 EU}{\partial t \partial q} = (-\frac{\partial^2 p_I}{\partial \tau \partial q})[U(R_s, A+\gamma L+ct) - U(R_f, A+ct)] + \frac{\partial p_I}{\partial q}[\frac{\partial U(R_s,A+\gamma L+ct)}{\partial Y} - \frac{\partial U(R_f,A+ct)}{\partial Y}]c$, $\frac{\partial^2 EU}{\partial t \partial x_I} = (-\frac{\partial^2 p_I}{\partial \tau \partial x_I})[U(R_s, A+\gamma L+ct) - U(R_f, A+ct)] + \frac{\partial p_I}{\partial x_I}[\frac{\partial U(R_s,A+\gamma L+ct)}{\partial Y} - \frac{\partial U(R_f,A+ct)}{\partial Y}]c - \frac{\partial p_I}{\partial \tau}[\frac{\partial U(R_s,A+\gamma L+ct)}{\partial R}]R_s' + p_I[\frac{\partial^2 U(R_s,A+\gamma L+ct)}{\partial Y \partial R}]R_s'$, $\frac{\partial^2 EU}{\partial t \partial A} = (-\frac{\partial p_I}{\partial \tau})[\frac{\partial U(R_s,A+\gamma L+ct)}{\partial Y} - \frac{\partial U(R_f,A+ct)}{\partial Y}] + [p_I\frac{\partial^2 U(R_s,A+\gamma L+ct)}{\partial Y^2} + (1-p_I)\frac{\partial^2 U(R_s,A+\gamma L+ct)}{\partial Y^2}]c = \frac{\partial^2 EU}{\partial t \partial c}/t$, and $\frac{\partial^2 EU}{\partial t \partial L} = (\frac{L}{\gamma})\frac{\partial^2 EU}{\partial t \partial \gamma} = [(-\frac{\partial p_I}{\partial \tau})\frac{\partial U(R_s,A+\gamma L+ct)}{\partial Y} + p_I\frac{\partial^2 U(R_s,A+\gamma L+ct)}{\partial Y^2}c]\gamma$, all of which are generally ambiguous in sign, whereas $\frac{\partial^2 EU}{\partial t \partial \beta} = \frac{\partial^2 EU}{\partial t \partial x_O} = \frac{\partial^2 EU}{\partial t \partial K_O} = 0$. Next note that $\frac{\partial^2 E\Pi}{\partial c \partial t} = [\frac{\partial^2 p_O}{\partial e_O \partial \tau} + \frac{\partial^2 p_O}{\partial e_O^2}(c)](t\pi) + (\frac{\partial p_O}{\partial e_O}\pi - 1) < 0$ at an interior solution to the firm's problem, $\frac{\partial^2 E\Pi}{\partial c \partial G} = \frac{\partial^2 p_O}{\partial e_O^2}\beta t\pi < 0$, $\frac{\partial^2 E\Pi}{\partial c \partial F} = \frac{\partial^2 p_O}{\partial e_O^2}t\pi < 0$, $\frac{\partial^2 E\Pi}{\partial c \partial \alpha} = \frac{\partial^2 E\Pi}{\partial c \partial x_I} = \frac{\partial^2 E\Pi}{\partial c \partial K_I} = \frac{\partial^2 E\Pi}{\partial t \partial A} = \frac{\partial^2 EU}{\partial c \partial L} = 0$, $\frac{\partial^2 E\Pi}{\partial c \partial \beta} = \frac{\partial^2 p_O}{\partial e_O^2}Gt\pi < 0$, $\frac{\partial^2 E\Pi}{\partial c \partial q} = \frac{\partial^2 p_O}{\partial e_O \partial q}t\pi > 0$, $\frac{\partial^2 E\Pi}{\partial c \partial x_O} = \frac{\partial^2 p_O}{\partial e_O \partial x_O}t\pi < 0$, and $\frac{\partial^2 E\Pi}{\partial c \partial K_O} = \frac{\partial^2 p_O}{\partial e_O^2}t\pi < 0$.

From the proof of the Theorem 3, her best reply is positively sloped if and only if $\frac{\partial^2 EU}{\partial t \partial c} > 0$, which occurs only if $\frac{\partial U(R_s,A+\gamma L+ct)}{\partial Y} \leq \frac{\partial U(R_f,A+ct)}{\partial Y}$ and she is either risk neutral or not too risk averse. From the expressions above, this implies that $\frac{\partial^2 EU}{\partial t \partial j} < 0$ for $j = G,F,\alpha,q,K_I,A,L,\gamma$, though the sign of $\frac{\partial^2 EU}{\partial t \partial x_I}$ is ambiguous. Further, $\frac{\partial^2 EU}{\partial t \partial \beta} = \frac{\partial^2 EU}{\partial t \partial x_O} = \frac{\partial^2 EU}{\partial t \partial K_O} = 0$, $\frac{\partial^2 E\Pi}{\partial c \partial \beta} < 0$, $\frac{\partial^2 E\Pi}{\partial c \partial x_O} < 0$, and $\frac{\partial^2 E\Pi}{\partial c \partial K_O} < 0$.

Conversely, her best reply is negatively sloped if and only if $\frac{\partial^2 EU}{\partial t \partial c} < 0$. In this case, $\frac{\partial^2 EU}{\partial t \partial \beta} = \frac{\partial^2 EU}{\partial t \partial x_O} = \frac{\partial^2 EU}{\partial t \partial K_O} = 0$, $\frac{\partial^2 E\Pi}{\partial c \partial \beta} < 0$, $\frac{\partial^2 E\Pi}{\partial c \partial x_O} < 0$, and $\frac{\partial^2 E\Pi}{\partial c \partial K_O} < 0$ as well. If, in addition, $\frac{\partial U(R_s,A+\gamma L+ct)}{\partial Y} \leq \frac{\partial U(R_f,A+ct)}{\partial Y}$, then $\frac{\partial^2 EU}{\partial t \partial G} = (-\frac{\partial^2 p_I}{\partial \tau \partial e_I})[U(R_s, A+\gamma L + ct) - U(R_f, A + ct)] + \frac{\partial p_I}{\partial e_I}[\frac{\partial U(R_s,A+\gamma L+ct)}{\partial Y} - \frac{\partial U(R_f,A+ct)}{\partial Y}]c < 0$ because $\frac{\partial^2 p_I}{\partial \tau \partial e_I} > 0$, $U(R_s, A + \gamma L + ct) > U(R_f, A + ct)$, and $\frac{\partial^2 p_I}{\partial e_I} > 0$. Similarly, $\frac{\partial^2 EU}{\partial t \partial j} < 0$ for $j = F,\alpha,q,K_I,A,L,\gamma$, though $\frac{\partial^2 EU}{\partial t \partial x_I}$ remains ambiguous.

The statements (i), (ii), (iiia,b), and (iiic1) in Theorem 4 then follow immediately.

Finally, for $j = G, F$, $\frac{\partial^2 EU}{\partial t \partial j} < 0$ if her best reply is negatively sloped and $\frac{\partial^2 E\Pi}{\partial c \partial j} < 0$, so $\frac{\partial t^*}{\partial j} + \frac{\partial c^*}{\partial j} = \{\frac{\partial^2 EU}{\partial t \partial j}[\frac{\partial^2 E\Pi}{\partial c \partial t} - \frac{\partial^2 E\Pi}{\partial c^2}] + [\frac{\partial^2 EU}{\partial t \partial c} - \frac{\partial^2 EU}{\partial t^2}]\frac{\partial^2 E\Pi}{\partial c \partial j}\}/D_2 > 0$ if and only if both $\frac{\partial^2 E\Pi}{\partial c \partial t} - \frac{\partial^2 E\Pi}{\partial c^2} < 0$ and $\frac{\partial^2 EU}{\partial t \partial c} - \frac{\partial^2 EU}{\partial t^2} < 0$, which contradicts the local stability condition and proves statement (iiic2).

VI. The proof of Theorem 5 is analogous to that of Theorem 1.

VII. Proof of Theorem 6

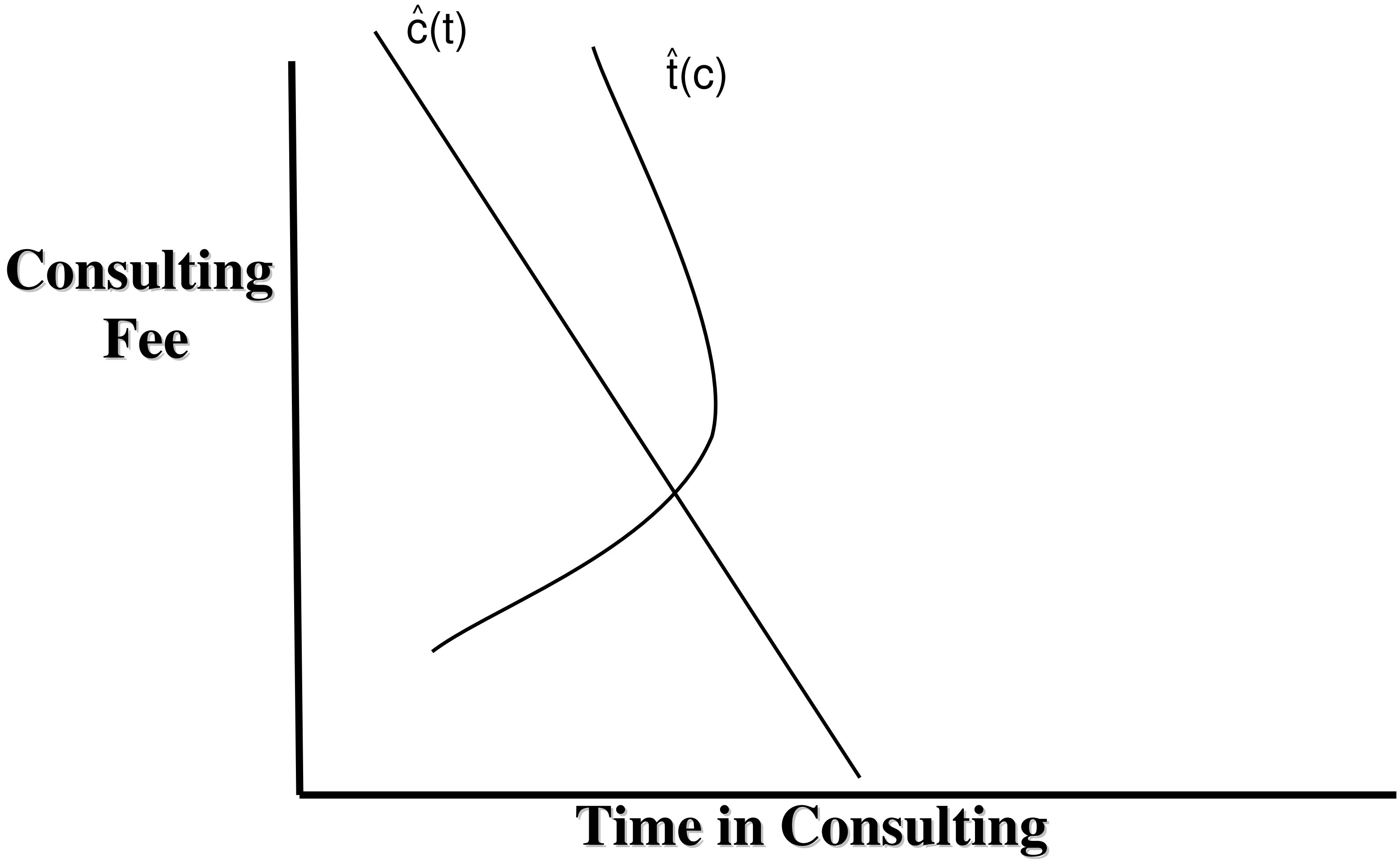
Set $\Theta = -\frac{\partial^2 p_I}{\partial \tau^2}\frac{\partial t^*}{\partial G} + \frac{\partial^2 p_I}{\partial \tau \partial e_I}$, $\Lambda = \frac{\partial^2 p_I}{\partial e_I^2} - \frac{\partial^2 p_I}{\partial \tau \partial e_I}\frac{\partial t^*}{\partial G}$, $\Phi = -\frac{\partial^2 p_I}{\partial \tau^2}\frac{\partial t^*}{\partial F} + \frac{\partial^2 p_I}{\partial \tau \partial e_I}\alpha$, $\Omega = \frac{\partial^2 p_I}{\partial e_I^2}\alpha - \frac{\partial^2 p_I}{\partial \tau \partial e_I}\frac{\partial t^*}{\partial F}$, and $\Delta U_g = U_g(R_{gs}) - U_g(R_{gf}) > 0$. Recall from (12) $\Theta > 0$, $\Lambda > 0$, $\Phi > 0$, and $\Omega > 0$ by assumption. Then $\frac{\partial^2 P_g}{\partial G \partial F} = [\Theta(-\frac{\partial t^*}{\partial F}) + \frac{\partial p_I}{\partial \tau}(-\frac{\partial^2 t^*}{\partial G \partial F}) + \Lambda\alpha]\Delta U_g > 0$ because $\frac{\partial t^*}{\partial F} < 0$ and $\frac{\partial^2 t^*}{\partial F \partial G} \approx 0$ by assumption. Similarly, $\frac{\partial^2 P_f}{\partial F \partial G} = [\Phi(-\frac{\partial t^*}{\partial G}) - \frac{\partial p_I}{\partial \tau}\frac{\partial^2 t^*}{\partial G \partial F} + \Omega](\pi_I - L) + \{[\frac{\partial^2 p_O}{\partial \tau^2}(\frac{\partial t^*}{\partial G}) + \frac{\partial^2 p_O}{\partial \tau \partial e_O}(\frac{\partial e_O^*}{\partial G})]\frac{\partial t^*}{\partial F} + \frac{\partial p_O}{\partial \tau}\frac{\partial^2 t^*}{\partial G \partial F}\}\pi > 0$ because $\frac{\partial t^*}{\partial G} < 0$, $\frac{\partial t^*}{\partial F} < 0$, $\frac{\partial^2 t^*}{\partial F \partial G} \approx 0$, and $\frac{\partial^2 p_O}{\partial \tau^2}(\frac{\partial t^*}{\partial G}) + \frac{\partial^2 p_O}{\partial \tau \partial e_O}(\frac{\partial e_O^*}{\partial G}) \leq 0$ by the hypothesis of the theorem.

VIII. Proof of Theorem 7 and Corollary 8.

In this case, we have $\frac{\partial G^*}{\partial j} = [\frac{\partial^2 P_g}{\partial G \partial F}\frac{\partial^2 P_f}{\partial F \partial j} - \frac{\partial^2 P_f}{\partial F^2}\frac{\partial^2 P_g}{\partial G \partial j}]/D_1$ and $\frac{\partial F^*}{\partial j} = [\frac{\partial^2 P_f}{\partial F \partial G}\frac{\partial^2 P_g}{\partial G \partial j} - \frac{\partial^2 P_g}{\partial G^2}\frac{\partial^2 P_f}{\partial F \partial j}]/D_1$, for $j = \alpha,\beta,q,x_I,K_I,x_O,K_O,\gamma,A,L$, where $D_1 = \frac{\partial^2 P_g}{\partial G^2}\frac{\partial^2 P_f}{\partial F^2} - \frac{\partial^2 P_g}{\partial G \partial F}\frac{\partial^2 P_f}{\partial F \partial G} > 0$ by the assumption that the equilibrium is locally stable, and where $\frac{\partial^2 P_g}{\partial G \partial F} > 0$ and $\frac{\partial^2 P_f}{\partial F \partial G} > 0$ from the preceding theorem. First, observe that $\frac{\partial^2 P_g}{\partial G \partial j} = [\Theta(-\frac{\partial t^*}{\partial j}) - \frac{\partial p_I}{\partial \tau}\frac{\partial^2 t^*}{\partial G \partial j}]\Delta U_g > 0$ for $j = \gamma,L$ because $\frac{\partial t^*}{\partial j} < 0$ for $j = \gamma,L$ and $\frac{\partial^2 t^*}{\partial i \partial j} \approx 0$, $\frac{\partial^2 P_g}{\partial G \partial j} = [\Theta(-\frac{\partial t^*}{\partial j}) - \frac{\partial p_I}{\partial \tau}\frac{\partial^2 t^*}{\partial G \partial j} + \Lambda]\Delta U_g > 0$ for $j = \alpha,K_I$ because $\frac{\partial t^*}{\partial j} < 0$ for $j = \alpha,K_I$, but $\frac{\partial^2 P_g}{\partial G \partial j} = [\Theta(-\frac{\partial t^*}{\partial j}) - \frac{\partial p_I}{\partial \tau}\frac{\partial^2 t^*}{\partial G \partial j}]\Delta U_g$ for $j = \beta,x_O,K_O$ , $\frac{\partial^2 P_g}{\partial G \partial q} = [\Theta(-\frac{\partial t^*}{\partial q}) - \frac{\partial p_I}{\partial \tau}\frac{\partial^2 t^*}{\partial G \partial q} - \frac{\partial^2 p_I}{\partial \tau \partial q}\frac{\partial t^*}{\partial G} + \frac{\partial^2 p_I}{\partial e_I \partial q}]\Delta U_g$ and $\frac{\partial^2 P_g}{\partial G \partial x_I} = [\Theta(-\frac{\partial t^*}{\partial x_I}) - \frac{\partial p_I}{\partial \tau}\frac{\partial^2 t^*}{\partial G \partial x_I} - \frac{\partial^2 p_I}{\partial \tau \partial x_I}\frac{\partial t^*}{\partial G} + \frac{\partial^2 p_I}{\partial e_I \partial x_I}]\Delta U_g + \Theta U_g' R_{sg}'$ are all ambiguous. Similarly, $\frac{\partial^2 P_f}{\partial F \partial j} = [\Phi(-\frac{\partial t^*}{\partial j}) - \frac{\partial p_I}{\partial \tau}\frac{\partial^2 t^*}{\partial F \partial j}](\pi_I - L) + [\frac{\partial^2 p_O}{\partial \tau^2}\frac{\partial t^*}{\partial j} + \frac{\partial^2 p_O}{\partial \tau \partial e_O}\frac{\partial e_O^*}{\partial j}]\frac{\partial t^*}{\partial F}\pi + \frac{\partial p_O}{\partial \tau}\frac{\partial^2 t^*}{\partial F \partial j}\pi > 0$ for $j = \gamma,L$, because $\frac{\partial t^*}{\partial j} < 0 < \frac{\partial c^*}{\partial j}$ for these $j$ and $\frac{\partial^2 p_O}{\partial \tau^2}(\frac{\partial t^*}{\partial j}) + \frac{\partial^2 p_O}{\partial \tau \partial e_O}(\frac{\partial e_O^*}{\partial j}) \leq 0$ by the hypothesis of the theorem. Next note that $\frac{\partial^2 P_f}{\partial F \partial K_I} = [\Phi(-\frac{\partial t^*}{\partial K_I}) + \Omega - \frac{\partial p_I}{\partial \tau}\frac{\partial^2 t^*}{\partial F \partial K_I}](\pi_I - L) + \{[\frac{\partial^2 p_O}{\partial \tau^2}\frac{\partial t^*}{\partial j} + \frac{\partial^2 p_O}{\partial \tau \partial e_O}\frac{\partial e_O^*}{\partial K_I}]\frac{\partial t^*}{\partial F} + \frac{\partial p_O}{\partial \tau}\frac{\partial^2 t^*}{\partial F \partial j}\}\pi > 0$ because $\frac{\partial t^*}{\partial K_I} < 0$ and $\frac{\partial^2 p_O}{\partial \tau^2}\frac{\partial t^*}{\partial K_I} + \frac{\partial^2 p_O}{\partial \tau \partial e_O}\frac{\partial e_O^*}{\partial K_I} \leq 0$ is assumed. However, $\frac{\partial^2 P_f}{\partial F \partial j} = [\Phi(-\frac{\partial t^*}{\partial j}) - \frac{\partial p_I}{\partial \tau}\frac{\partial^2 t^*}{\partial F \partial j}](\pi_I - L) + \{[\frac{\partial^2 p_O}{\partial \tau^2}\frac{\partial t^*}{\partial j} + \frac{\partial^2 p_O}{\partial \tau \partial e_O}\frac{\partial e_O^*}{\partial j}]\frac{\partial t^*}{\partial F} + \frac{\partial p_O}{\partial \tau}\frac{\partial^2 t^*}{\partial F \partial j}\}\pi$, $\frac{\partial^2 P_f}{\partial F \partial q} = [\Phi(-\frac{\partial t^*}{\partial q}) + \frac{\partial^2 p_I}{\partial e_I \partial q}\alpha - \frac{\partial^2 p_I}{\partial \tau \partial q}\frac{\partial t^*}{\partial F} - \frac{\partial p_I}{\partial \tau}\frac{\partial^2 t^*}{\partial F \partial q}](\pi_I - L) + \{[\frac{\partial^2 p_O}{\partial \tau^2}\frac{\partial t^*}{\partial q} + \frac{\partial^2 p_O}{\partial \tau \partial e_O}\frac{\partial e_O^*}{\partial q} + \frac{\partial^2 p_O}{\partial \tau \partial q}]\frac{\partial t^*}{\partial F} + \frac{\partial p_O}{\partial \tau}\frac{\partial^2 t^*}{\partial F \partial q}\}\pi$, $\frac{\partial^2 P_f}{\partial F \partial x_I} = [\Phi(-\frac{\partial t^*}{\partial x_I}) + \frac{\partial^2 p_I}{\partial e_I \partial x_I}\alpha - \frac{\partial^2 p_I}{\partial \tau \partial x_I}\frac{\partial t^*}{\partial F} - \frac{\partial p_I}{\partial \tau}\frac{\partial^2 t^*}{\partial F \partial x_I}](\pi_I - L) + \{[\frac{\partial^2 p_O}{\partial \tau^2}\frac{\partial t^*}{\partial x_I} + \frac{\partial^2 p_O}{\partial \tau \partial e_O}\frac{\partial e_O^*}{\partial x_I}]\frac{\partial t^*}{\partial F} + \frac{\partial p_O}{\partial \tau}\frac{\partial^2 t^*}{\partial F \partial x_I}\}\pi$, $\frac{\partial^2 P_f}{\partial F \partial \alpha} = [\Phi(-\frac{\partial t^*}{\partial \alpha}) + \Omega F + \frac{\partial p_I}{\partial e_I} - \frac{\partial p_I}{\partial \tau}\frac{\partial^2 t^*}{\partial F \partial \alpha}](\pi_I - L) + \{[\frac{\partial^2 p_O}{\partial \tau^2}\frac{\partial t^*}{\partial \alpha} + \frac{\partial^2 p_O}{\partial \tau \partial e_O}\frac{\partial e_O^*}{\partial \alpha}]\frac{\partial t^*}{\partial F} + \frac{\partial p_O}{\partial \tau}\frac{\partial^2 t^*}{\partial F \partial \alpha}\}\pi$, and $\frac{\partial^2 P_f}{\partial F \partial x_O} = [\Phi(-\frac{\partial t^*}{\partial x_O}) - \frac{\partial p_I}{\partial \tau}\frac{\partial^2 t^*}{\partial F \partial x_O}](\pi_I - L) + \{[\frac{\partial^2 p_O}{\partial \tau^2}\frac{\partial t^*}{\partial x_O} + \frac{\partial^2 p_O}{\partial \tau \partial e_O}\frac{\partial e_O^*}{\partial x_O} + \frac{\partial^2 p_O}{\partial \tau \partial x_O}]\frac{\partial t^*}{\partial F} + \frac{\partial p_O}{\partial \tau}\frac{\partial^2 t^*}{\partial F \partial x_O}\}\pi$ are all ambiguous. The statements of the theorem then follow immediately from these results plus locally stability.

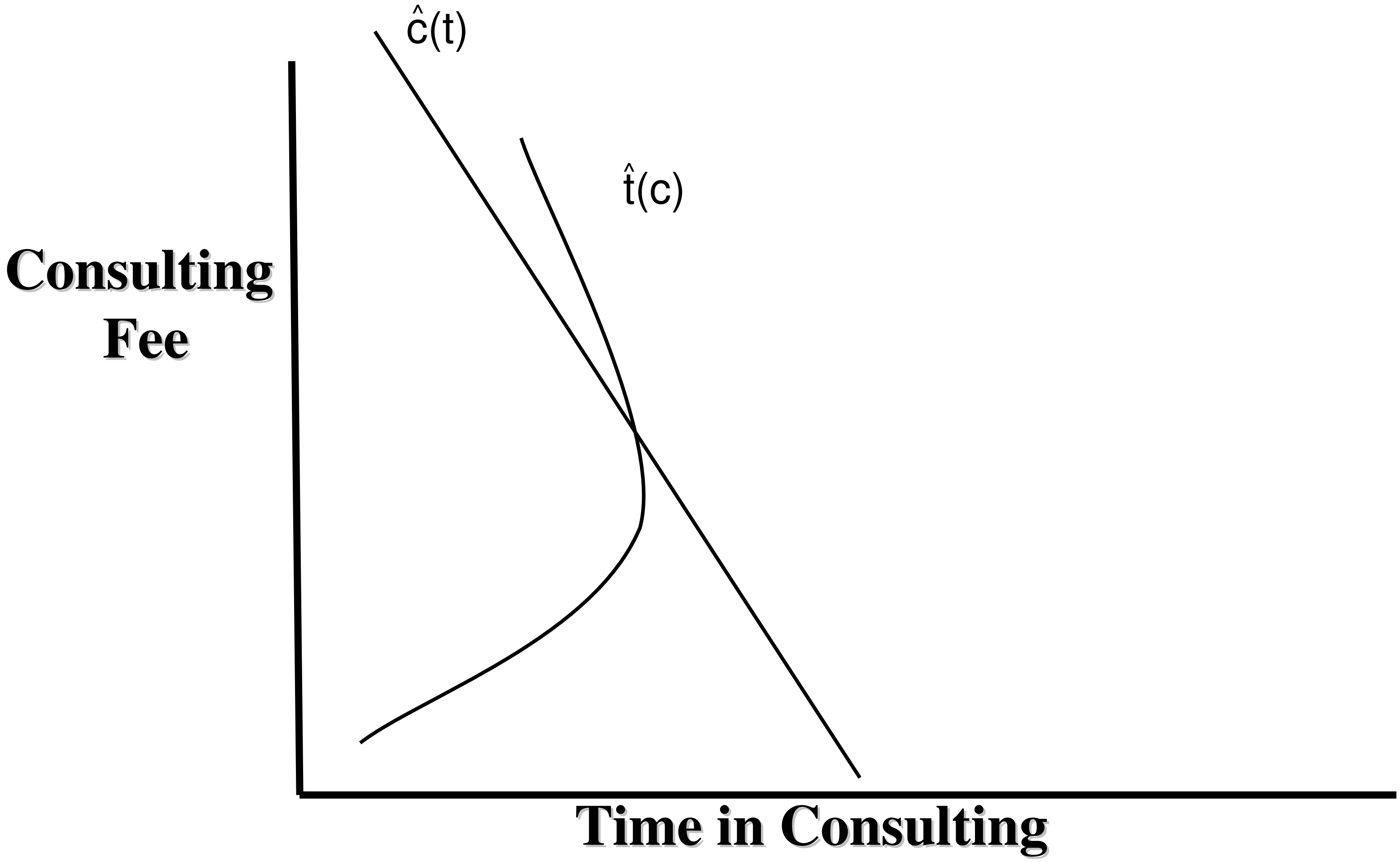
Finally, from the preceding, and ignoring second stage effects, we have $\frac{\partial^2 P_g}{\partial G \partial j} = 0$ for $j = \beta,x_O,K_O$ and $\frac{\partial^2 P_g}{\partial G \partial q} = \frac{\partial^2 p_I}{\partial e_I \partial q}\Delta U_g > 0$, but $\frac{\partial^2 P_g}{\partial G \partial x_I} = \frac{\partial^2 p_I}{\partial e_I \partial x_I}\Delta U_g +$

$\frac{\partial^2 p_I}{\partial\tau\partial e_I}U'_g R'_{sg}$ remains ambiguous, while $\frac{\partial^2 P_f}{\partial F\partial\beta} = \frac{\partial^2 p_O}{\partial e_O^2}G^*\pi < 0$, $\frac{\partial^2 P_f}{\partial F\partial x_O} = \frac{\partial^2 p_I}{\partial e_I\partial x_O}\pi < 0$, and $\frac{\partial^2 P_f}{\partial F\partial K_O} = \frac{\partial^2 p_O}{\partial e_O^2}\pi < 0$ , $\frac{\partial^2 P_f}{\partial F\partial x_I} = \frac{\partial^2 p_I}{\partial e_I\partial x_I}\alpha(\pi_I - L) < 0$, and $\frac{\partial^2 P_f}{\partial F\partial q} = \frac{\partial^2 p_I}{\partial e_I\partial q}\alpha(\pi_I - L) > 0$.



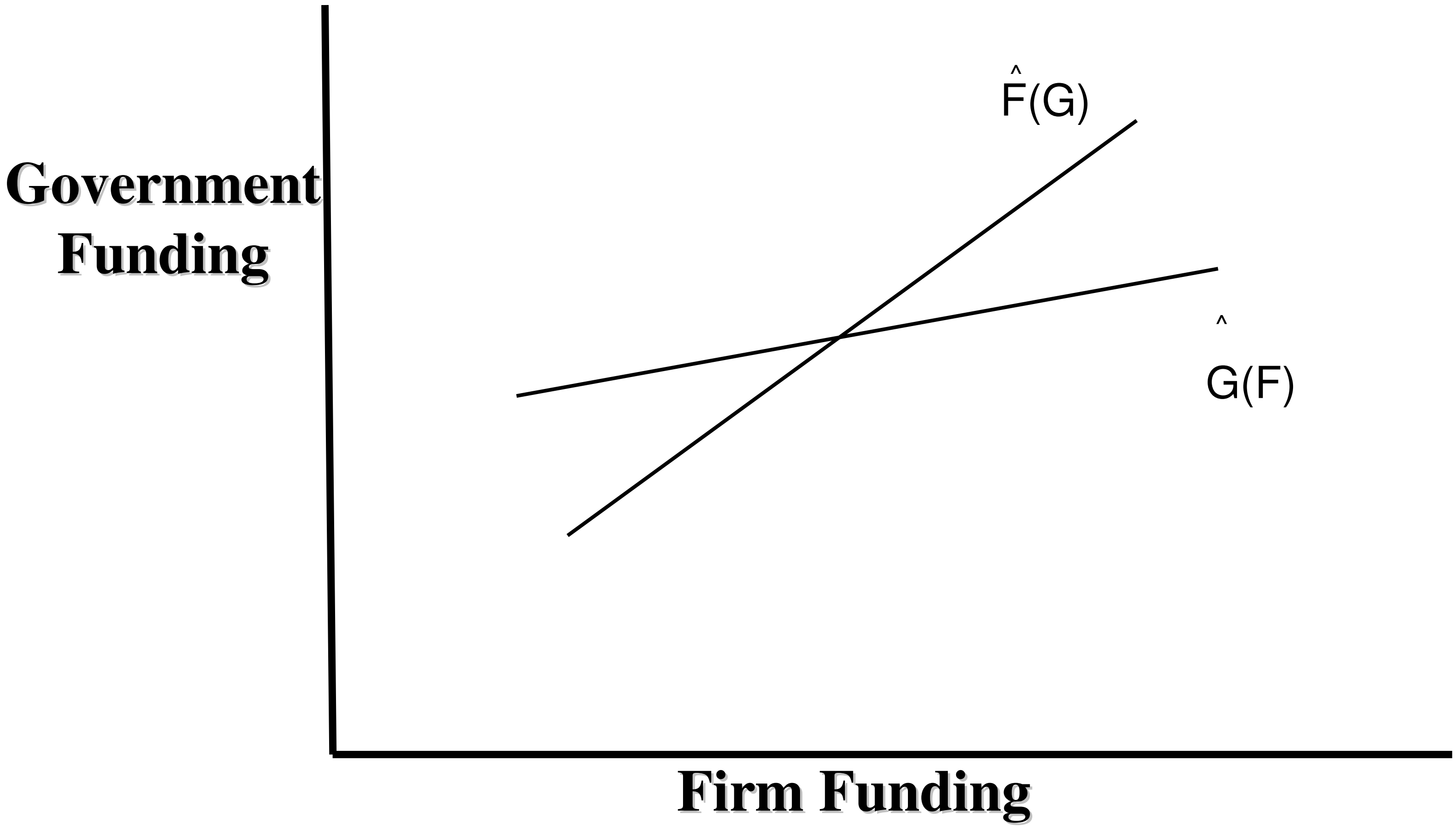

**Second Stage**

**Figure 1**




**Second Stage**

**Figure 2**




**First Stage**

**Figure 3**

**Table 1. Summary Statistics**

| | | No. Obs. | Mean | SE | Min | Max |
|---|---|---|---|---|---|---|
| PUBS | Annual publications | 1690 | 7.27 | 8.49 | 0.00 | 51.00 |
| PUB_CITES | Citations to publications | 1690 | 269.95 | 558.41 | 0.00 | 6557.00 |
| AGE | Inventor age | 1631 | 49.02 | 9.98 | 28.00 | 83.00 |
| GOV_FND | Federal funding | 1690 | 0.79 | 1.78 | 0.00 | 15.02 |
| IND_FND | Industrial funding | 1690 | 0.16 | 0.54 | 0.00 | 4.18 |
| URBAN | University in urban area | 1690 | 0.64 | 0.48 | 0.00 | 1.00 |
| PAT_CITES | Patent citations to prior patents | 1690 | 11.94 | 19.72 | 0.00 | 354.00 |
| ORIGINAL | Measure of originality | 1526 | 0.47 | 0.29 | 0.00 | 0.93 |
| INVENTSHARE | Inventor share of licensing revenue | 1690 | 30.86 | 6.63 | 20.00 | 52.00 |
| FOR_CITES | Forward patent citations | 1690 | 20.91 | 35.81 | 0.00 | 459.00 |
| LAB_SIZE | Lab size | 1690 | 12.79 | 42.21 | 0.00 | 418.84 |
| ARTICLE_CITES | Articles cited in patent | 1690 | 16.77 | 23.09 | 0.00 | 203.00 |
| MALE | Inventor is male | 1659 | 0.95 | 0.22 | 0.00 | 1.00 |
| UNIVASSIGN | Patent is assigned to university | 1690 | 0.70 | 0.46 | 0.00 | 1.00 |
| PUBLIC | University is public | 1690 | 0.33 | 0.47 | 0.00 | 1.00 |

**Table 2. Consulting**

Dependent Variable: ASSIGN=1 if assigned to a university.

| | Part A | | | Part B | | |
|---|---|---|---|---|---|---|
| | Odds Ratio | t-Statistic | | Odds Ratio | t-Statistic | |
| LOG_LAG_PUB | 1.627 | 3.87 | *** | 1.487 | 2.96 | *** |
| LOG_LAG_PUB_CITES | 0.920 | -1.61 | | 0.969 | -0.58 | |
| LOG_LAG_GOVFND | 0.973 | -4.08 | *** | 0.984 | -2.27 | ** |
| LOG_LAG_INDFND | 1.025 | 3.71 | *** | 1.020 | 2.45 | ** |
| ENG | 2.002 | 3.34 | *** | 2.372 | 3.99 | *** |
| PHY_SIC | 0.767 | -1.29 | | 0.845 | -0.78 | |
| ORIGINAL | 2.817 | 3.66 | *** | 2.659 | 3.28 | *** |
| LOG_PATENT_CITES | 0.438 | -8.61 | *** | 0.445 | -8.15 | *** |
| LOG_FOR_CITES | 0.908 | -1.61 | | 0.988 | -0.20 | |
| LOG_ARTICLE_CITES | 1.452 | 6.81 | *** | 1.420 | 6.07 | *** |
| AGE | 1.151 | 2.41 | ** | 1.121 | 1.81 | * |
| AGESQ | 0.999 | -2.39 | ** | 0.999 | -1.72 | * |
| MALE | 0.960 | -0.11 | | 1.208 | 0.45 | |
| PUBLIC | 0.710 | -1.37 | | | | |
| INVENTSHARE | 1.015 | 1.14 | | | | |
| URBAN | 0.612 | -2.61 | *** | | | |
| University Fixed Effets | NO | | | YES | | |
| R-Square | 0.129 | | | 0.165 | | |
| Observations | 1312 | | | 1312 | | |

**Table 3. Federal Funding**

| | Part a Coeff | t-Statistic | | Part b Coeff | t-Statistic | |
|---|---|---|---|---|---|---|
| LOG_INDFND | 0.015 | 1.49 | | 0.022 | 2.18 | ** |
| LOG_LAG_GOVFND | 0.158 | 14.40 | *** | 0.124 | 10.98 | *** |
| LOG_LAG_PUB | 0.048 | 0.51 | | 0.123 | 1.39 | |
| LOG_LAG_PUB_CITES | 0.069 | 1.69 | * | 0.036 | 0.93 | |
| LAB_SIZE | 0.082 | 1.58 | | 0.053 | 1.09 | |
| SPILL | -0.073 | -1.14 | | 0.004 | 0.07 | |
| ENG | -0.157 | -1.33 | | -0.237 | -2.07 | ** |
| PHY_SIC | -0.027 | -0.21 | | 0.027 | 0.22 | |
| MALE | 0.347 | 1.54 | | 0.282 | 1.34 | |
| AGE | 0.013 | 0.32 | | 0.083 | 2.13 | ** |
| AGESQ | 0.000 | -0.40 | | -0.001 | -2.21 | ** |
| INVENTSHARE | -0.012 | -1.33 | | | | |
| University Fixed Effets | NO | | | YES | | |
| Observations | 1044 | | | 1044 | | |

**Table 4. Industry Funding**

| | Part a Coeff | t-Statistic | | Part b Coeff | t-Statistic | |
|---|---|---|---|---|---|---|
| LOG_GOVFND | 0.037 | 2.26 | ** | 0.047 | 1.89 | * |
| LOG_LAG_INDFND | 0.091 | 7.14 | *** | 0.099 | 7.44 | *** |
| LOG_LAG_PUB | 0.194 | 1.24 | | 0.168 | 1.06 | |
| LOG_LAG_PUB_CITES | -0.093 | -1.3 | | -0.084 | -1.19 | |
| LAB_SIZE | 0.210 | 2.74 | *** | 0.183 | 2.37 | ** |
| SPILL | -0.595 | -4.43 | *** | -0.539 | -3.86 | *** |
| ENG | 0.398 | 2.01 | ** | 0.506 | 2.40 | ** |
| PHY_SIC | -0.171 | -0.74 | | -0.054 | -0.23 | |
| MALE | 0.385 | 1.11 | | 0.159 | 0.46 | |
| AGE | 0.063 | 0.94 | | 0.087 | 1.31 | |
| AGESQ | -0.001 | -0.96 | | -0.001 | -1.30 | |
| INVENTSHARE | -0.008 | -0.61 | | | | |
| University Fixed Effets | NO | | | YES | | |
| Observations | 1044 | | | 1044 | | |